	2.2 Runge-Kutta Method of Approximation	We can generalize the concept of a linear equation to a linear differential	4.2 Steps for Solving Nonhomogeneous Linear Equa-	5.2.1 Example	
$y' = 3t^2(1 + y) \rightarrow \frac{dy}{v} = 3t^2(1 + y)$	If we have an IVP, we can calculate the next values with a process similar	equation. A differential equation $F(y, y', y'', \dots, y^n) = f(t)$ is linear if it is in the form: $a_n(t)\frac{d^ny}{dt^n} + a_{n-1}(t)\frac{d^{-1}y}{dt^{n-1}} + \dots + a_1(t)\frac{d^1y}{dt^1} + a_0(t)\frac{d^ny}{dt^0} = f(t)$ where		$\frac{dy}{dt} - y = t$	If $x(t)$ is the amount of dissolved substance, then $\frac{dx}{tt} = \text{Rate In} - \text{Rate Out}$ (c)
	to (11)	all function of t are assumed to be defined over some common interval I. If $f(t) = 0$ over the interval I, the differential equation is said to be	1. Find all \mathbf{u}_n of $L(\mathbf{u}) = 0$.	$\mu(t) = e^{\int -1 dt} \rightarrow e^{-t}$	$\frac{1}{dt} = \text{Kate In} - \text{Kate Out} (8)$ Where $\int \text{Rate In} = \text{Concentration in} \cdot \text{Flow Rate In}$
$\frac{dy}{1+y} = 3t^2 \rightarrow \int \frac{dy}{1+y} = \int 3t^2$	$\begin{cases} t_{n+1} = t_n + h \\ y_{n+1} = y_n + hk_{n2} \end{cases}$	If f(i) = 0 over the interval r, the universital equation is said to be homogeneous. We will also introduce some easier notation for linear algebraic equa.	2. Γ ma any $\mathbf{u}_p o f L(\mathbf{u}) = f$.	$e^{-t}y = \int te^{-t} dt \rightarrow e^{-t}(-t - 1) + c$	Where Rate Out = Concentration in · Flow Rate In Rate Out = Concentration in · Flow Rate Out
$\ln 1 + y = t^3 + c \rightarrow 1 + y = e^c e^{t^3}$ $y = ce^{t^3} - 1, k \neq 0$	Where (2)	tions: $\vec{x} = [x_1, x_2,, x_n]$ and for linear differential equations: $\vec{y} =$	 Add them, u = u_n + u_p to get all solutions of L(u) = f. 	$y(t) = ce^{t} - t - 1$	We can also use these for cooling problems. Newton's law of cooling is as follows
	$k_{n1} = f(t_n, y_n)$ $k_{n2} = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_{n1}\right)$	$[y^n, y^{n-1}, \dots, y', y]$ We will also introduce the linear operator L	5 Solving 1 st Order Linear Differential Equa-		
2 Approximation Methods	$\kappa_{n2} = \int \left(l_n + \frac{1}{2}, y_n + \frac{1}{2} \kappa_{n1} \right)$ For more precision, use the fourth order Runge-Kutta method. It is the	$L(\vec{x}) = a_1x_1 + a_2x_2 + \cdots + a_nx_n$ $L(\vec{y}) = a_n(t)\frac{d^ny}{dtn} + a_{n-1}(t)\frac{d^{n-1}y}{dtn-1} + \cdots + a_1(t)\frac{d^1y}{dt} + a_0(t)\frac{d^1y}{dt^0}$	tions	6 Applications of 1 st Order Linear Differen- tial Equations	$\frac{dT}{dt} = k(M - T)$ (9) (9)
2.1 Euler's Method (Tangent Line Method) - 1768	most commonly used method both because of its speed as well as its relative	$L(\mathbf{y}) = a_n(t) \frac{dt^n}{dt^n} + a_{n-1}(t) \frac{dt^{n-1}}{dt^{n-1}} + \dots + a_1(t) \frac{dt^1}{dt^1} + a_0(t) \frac{dt^0}{dt^0}$	5.1 Euler-Lagrange 2-Stage Method	-	Where $\begin{cases} T \rightarrow \text{Temperature of the Object} \\ M \rightarrow \text{Temperature of the Surroundings} \end{cases}$
With a given function $y' = f(t, y)$ and a given set point p_0 we can approx	precision.	4.1 Properties	To solve a linear differential equation in the form $y' + p(t)y = f(t)$ using this method:	6.1 Growth and Decay The function	7 Systems of Differential Equations
mate the line point by point.	$\begin{cases} t_{n+1} = t_n + h \\ y_{n+1} = y_n + \frac{h}{h} (k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}) \end{cases}$	A solution of the algebraic is any \vec{x} that satisfies the definition of linear algebraic equations, while a solution of the differential is for any \vec{y} that		$\frac{dy}{dt} = ky$	If one or more functions are dependent on other functions, then we call them
For the initial value problem $y' = f(t, y), y(t_0) = y_0$	Where	satisfies the definition of linear differential equations. For homogeneous linear equations:	1. Solve $y' + p(t)y = 0$ by separation of variables to get $y_n = ce^{-\int p(t) dt}$	can be called the growth or decay equation depending on the sign of k . We can explicitly find the solution to these equations:	Coupled $\int y' = xy$
Use the formulas $\begin{cases} t_{n+1} = t_n + h \\ y_{n+1} = y_n + hf(t_n, y_n) \end{cases}$ (1)	1) $k_{n1} = f(t_n, y_n)$ $k_{n2} = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_{n1}\right)$ (3)	 A constant multiple of a solution is also a solution. 	2. Solve $v'(t)e^{-\int p(t)dt}=f(t)$ for $v(t)$ to get the particular solution $y_p=v(t)e^{-\int p(t)dt}$	For each k , the solution of the IVP	condid Othermite and effective decondid
,	(/	 The sum of two solutions is also a solution. 	3. Combine to get	$\frac{dy}{dt} = ky, y(0) = y_0$ (3)	coupled. Otherwise we can them decoupled. $ \begin{aligned} & & \\$
2.1.1 Example	$k_{n3} = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_{n2}\right)$	Linear Operator Properties:	$y(t) = y_n + y_p = ce^{-\int p(t) dt} + e^{-\int p(t) dt} \int f(t)e^{\int p(t) dt} dt$ (4)	dl (6)	7.1 Autonomous First Order System
Obtain Euler approximation on $[0, 0.4]$ with step size 0.1 of y' = -2ty + t and $y(0) = -1$	$k_{n4} = f\left(t_n + h, y_n + hk_{n3}\right)$	 L(kũ) = kL(ũ), k ∈ ℝ. 	j	$y(t) = y_0 e^{kt}$	Autonomous systems are not dependent on t, so we can treat them a little
$h = 0.1, \begin{cases} t_0 = 0 \\ m = -1 \end{cases}$	3 Picard's Theorem	• $L(\vec{u} + \vec{w}) = L(\vec{u}) + L(\vec{w}).$	 5.2 Integrating Factor Method 1. Find the integrating factor μ(t) = e^{∫ p(t)dt} (Note, ∫ p(t) dt can be any 	We can use these equations for a wide variety of different equations such as continuously compounding interest:	differently. For these equations we can use a phase plane, vector field, and the trajectory of the solution.
$n = 0.1, y_0 = -1$ $t_1 = t_0 + h = 0.1$	Theorem 1 (Picard's). Suppose the function $f(t, y)$ is continuous on the region $R = \{(t, y) a < t < b, c < y < d\}$ and $(t_0, y_0) \in R$. Then there exists	4.1.1 Superposition Principle	antiderivative. In other words, don't bother with the addition of a	dA	The functions $x(t)$ and $y(t)$ can give us a parametric curve. This means that at any given point on the curve, we also have a tangent vector given
\rightarrow $\begin{cases} t_1 = t_0 + h = 0.1 \\ y_1 = y_0 + hf(t_0, y_0) = -1 \end{cases}$	a positive number h such that the IVP has a solution for t in the interval $(t_0 - h, t_0 + h)$. Furthermore, if $f_y(t, y)$ is also continuous on R , then that	Let \vec{u}_1 and \vec{u}_2 be any solutions of the homogeneous linear equation $L(\vec{u}) = 0$	constant.)	$A(t) = A_{c}e^{tt}$	by $\frac{dx}{dt}$ and $\frac{dx}{dt}$. Every solution of a system we call a state of the system, and the collection
\rightarrow $\begin{cases} t_2 = t_1 + h = 0.2 \end{cases}$	(i) - n, i) + n). Furthermore, if fy(t, y) is uso commuous on n, then that solution is unique.	Their sum is also a solution. A constant multiple is a solution for any constant k .	 Multiply each side by the integrating factor to get μ(t)(y' + p(t)y) = f(t)μ(t) Which will always reduce to d/dt (e^{f p(t) dt}y(t)) = f(t)e^{f p(t) dt} 	$A(t) = A_0 t$	of all the trajectories and states is called a phase portrait. An equilibrium point for this two dimensional system is an (x,y) point
$y_2 = y_1 + hf(t_1, y_1) = -0.97$ $\int t_3 = t_2 + h = 0.3$	4 Linearity and Nonlinearity		3. Take the antiderivative of both sides $e^{\int p(t) dt}y(t) = \int f(t)e^{\int p(t) dt}dt + c$	0.2 Mixing and cooling	where $\frac{dy}{dt} = 0 = \frac{dx}{dt}$
$y_3 = y_2 + hf(t_2, y_2) = -0.9112$	An equation $F(x, x_2, x_3,, x_n) = c$ is linear if it is in the form $a_1x_1 + a_2x_2 +$	4.1.2 Nonhomogeneous Principle Let \vec{u}_1 be any solution to a linear nonhomogeneous equation $L(\vec{u}) = c$	4. Solve for y	We can also use these models for mixing and cooling problems. A mixing problem consists of some amount of substance goes into a receptacle at a	7.2 Graphical Methods for Solving
\rightarrow $\begin{cases} t_4 = t_3 + h = 0.4 \\ y_4 = y_3 + hf(t_3, y_3) = -0.826528 \end{cases}$	$\cdots + a_n x_n = c$ where a_n are constants. Furthermore, if $c = 0$, the equation is said to be homogeneous.	(algebraic) or $L(\vec{u}) = f(t)$ (differential), then $\vec{u} = \vec{u}_n + \vec{u}_p$ is also a solution where \vec{u} is a solution to the associated homogeneous equation $L(\vec{u}) = 0$.	$y(t) = e^{\int p(t) dt} \int f(t)e^{\int p(t) dt} dt + ce^{\int p(t) dt}$ (5)	certain rate, and some amount of mixed substance comes out. We can model	Sketching is a pain in the assTherefore there are a couple tricks that we can use to make our lives easier.
			<u>,</u>		
We can use nullclines to more easily draw the solutions. Nullclines ar an adaptation of previously mentioned isoclines (??). A V nullcline is a	n	$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \end{bmatrix}$	 29.2 Elementary Row Operations Interchange row i and i B[*] = B, B[*] = B. 	9.5 Existence and Uniqueness 3 If the RREF has a row that looks like: $[0, 0, 0, \dots, 0 k]$ where k is a non-zero	59.7.2 Inverse Matrix by RREF For an $n \times n$ matrix A the following procedure either produces A^{-1} or
isocline of vertical slopes where $x' = 0$. An H nullcline is an isocline of horizontal slopes where $y' = 0$. Equilibria occurs at the point where these	e 8.1 Definitions	$\mathbf{A} = \begin{bmatrix} \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{bmatrix}$		constant, then the system has no solutions. We call this inconsistent. If the system has one or more solutions, we call it consistent.	proves that it's impossible.
two nullclines intersect. Note, when existence and uniqueness hold for an autonomous system		$\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n} \end{bmatrix}$	 Multiply row i by a constant. R[*]_i = cR_i 	In order to be unique, the system needs to be consistent.	1. Form the $n \times 2n$ matrix $M = [A I]$
phase plane trajectories never cross.	[a. a. a a.]	$\mathbf{B} = \begin{bmatrix} B_{21} & B_{22} & \cdots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix} (13)$	- Leaving j unto uched, add to i a constant times $j. \ R^*_i = R_i + cR_j$	 If every column is a pivot, the there is only one solution (unique solu- tion). 	
7.3 Quick Sketching Outline for Phase Portraits	$a_1 \ a_2 \ a_3 \ \cdots \ a_n$ $b_1 \ b_2 \ b_j \ \cdots \ b_n$ (41)	$\begin{bmatrix} B_{m1} & B_{m2} & \cdots & B_{mp} \end{bmatrix}$	These are handy when dealing with matrices and trying to obtain Reduced	 Else If most columns are pivots, there are multiple solutions (possibly 	 If the first n columns produce an Identity Matrix, then the last n are its inverse. Otherwise A is not invertible.
1. Nullclines and Equilibria	$\mathbf{A} = \begin{bmatrix} c_1 & c_2 & c_3 & \cdots & c_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix}$ (11)	$\mathbf{AB} = \begin{bmatrix} A_1 \cdot B_1 & A_2 \cdot B_1 & \cdots & A_3 \cdot B_1 \\ A_2 \cdot B_1 & A_2 \cdot B_2 & \cdots & A_2 \cdot B_3 \end{bmatrix}$	Row Echelon Form (??).	infinite).	
 Where x' = 0, slopes are vertical. Where y' = 0, slopes are horizontal. 	$\begin{bmatrix} m_1 & m_2 & m_3 & \cdots & m_n \end{bmatrix}$	$AB = \begin{bmatrix} \vdots & \vdots & \ddots & \vdots \\ A_m \cdot B_1 & A_m \cdot B_2 & \cdots & A_m \cdot B_n \end{bmatrix}$	9.3 Reduced Row Echelon Form	 Else the system is inconsistent. 	9.8 Invertibility and Solutions The matrix vector equation Ax = b where A is an n × n matrix has:
 where y = 0, slopes are norizontal. Where x' = y' = 0, we have equilibria. 		8.3 Matrix Transposition	$[\mathbf{A} \mathbf{b}] = \begin{bmatrix} 1 & 0 & 0 & b_1 \\ 0 & 1 & 0 & b_2 \end{bmatrix}$ (15)	9.6 Superposition, Nonhomogeneous Principle, and	 A unique solution x = A⁻¹b if and only if A is invertible.
2. Left-Right Directions	We can also describe these matrices by saying it has order $m \times n$ where m and n are the row and column sizes respectively. Two matrices are equal	We can flip a matrix disconally so that its columns become news and its	0 0 1 b3	RREF For any nonhomogeneous linear system $A\vec{x} = \vec{b}$, we can write the solutions	- Either no solutions or infinitely many solutions if ${\cal A}$ is not invertible.
 Where x' is positive, arrows point right. 	if they have the same m and n and the values contained are equal. We can also have matrices with orders $m \times 1$ or $n \times 1$ which are called column and	we can inp a matrix diagonaly so that its commiss become rows and its rows become columns. We call this the transpose of the matrix, written \mathbf{A}^T	 0 rows are at the bottom. 	as: $\vec{x} = \vec{x}_h + \vec{x}_p$ Where \vec{x}_h represents vectors in the set of homogeneous	For the homogeneous equation $A{\bf x}=0,$ there is always one solution, $x=0$
 Where x' is negative, arrows point left. 	row vectors.		 Leftmost non-zero entry is 1, also called the pivot (or leading 1). 	We can use RREF to find \vec{x}_{p} , and then, using the same RREF with \vec{b}	called the trivial solution. Let ${\bf A}$ be an $n\times n$ matrix. The following statements apply.
 Up-Down Directions Where y' is positive, arrows point up. 	If all entries are 0, we call it a zero matrix; however if all entries but the diagonal are zero, this is called an diagonal matrix. These diagonal num-	8.3.1 Properties • $(\mathbf{A}^T)^T = \mathbf{A}$		replaced by $\vec{0}$, find \vec{x}_h . The rank of a matrix r equals the number of pivot columns in the RREF.	 A is an invertible matrix.
 where y is positive, arrows point up. Where y' is negative, arrows point down. 	ber are called diagonal elements. A special diagonal matrix is the identity matrix, which is formed when the diagonal elements are ones.	$ (\mathbf{A}^{T})^{T} = \mathbf{A}^{T} + \mathbf{B}^{T} $	 Each pivot is further to the right than the one above. 	If r equals the number of variables, there is a unique solution. Otherwise if there is less, then it is not unique.	 A^T is an invertible matrix.
4. Check Uniqueness		 (kA)^T = kA^T for any scalar k. 	 Each pivot is the only non-zero entry in its column. 	9.7 Inverse of a Matrix	 A is row equivalent to I_n.
Where phase plane trajectories do not cross, we have uniqueness.	[1 0 0]	• $(\mathbf{A}\mathbf{B})^T = \mathbf{A}^T \mathbf{B}^T$	A less complete process gives us row echelon form, which allows for nonzero entries are allowed above the pivot.	When given a system of equations like: $\begin{cases} x + y = 1 \end{cases}$ we can rewrite it	 A has n pivot columns. The equation A\$\vec{x} = \$\vec{0}\$ has only the trivial solution, \$\vec{x} = \$\vec{0}\$.
7.4 Applications of Systems of Differential Equation	s $0 \ 1 \ \cdots \ 0$ $\vdots \ 1 \ \vdots$ (12)			4x + 5y = 6	 The equation AX = 0 has only the trivial solution, X = 0. The equation AX = 0 has a unique solution for every b in Rⁿ.
7.4.1 Predator-Prey Assumptions In the absence of foxes, the rabbit population will grow with the Malthusia	0 0 1	9 Matrices and Systems of Linear Equations	9.4 Gauss Jordan Reduction	in the form: $\begin{vmatrix} x \\ z \end{vmatrix} = \begin{vmatrix} z \\ z \end{vmatrix}$ For this sort of matrix, we can find	9.9 Determinants and Cramer's Rule
Growth Law: $\frac{dR}{dt} = a_R R$, $a_R > 0$ In the absence of rabbits, the fox populatio	n	9.1 Augmented Matrix	1. Given a system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$		9.9 Determinants and Cramer's Rule 9.9.1 2×2 Matrix
will die off according to the law: $\frac{dF}{dat} = -a_F F, a_F > 0$ When both foxe and rabbits are present, the number of interactions is \propto the product of th		An augmented matrix is where two different matrices are combined to form a new matrix.	2. Form augmented matrix [A b]	9.7.1 Properties	To find the determinant of a 2×2 matrix, the determinant is the diagonal
win the off according to the law: $\frac{1}{dat} = -a_F r_1 a_F > 0$ when both fixe and rabbits are present, the number of interactions is \propto the product of th population sizes, with inverse behavior. Thus we can get the Lotka-Volterr Equations for the predator prey model:		a new matrix. $\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \end{bmatrix}$	 Form augmented matrix [A b] Transform to RREF (??) using elementary row operations. 		products subtracted. This process is demonstrated below.
and rabbits are present, the number of interactions is \propto the product of th population sizes, with inverse behavior. Thus we can get the Lotka-Volterr Equations for the predator prey model: $\int \frac{da}{dt} = a_R R - c_R R F$	a ^a 8.2 Addition and Multiplication	a new matrix.	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ (16)
and rabbits are present, the number of interactions is \propto the product of th population sizes, with inverse behavior. Thus we can get the Lotka-Volterr Equations for the predator prey model:		a new matrix. $\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{11} & A_{12} & \cdots & A_{1m} \end{bmatrix} \begin{bmatrix} b_1 \\ b_1 \end{bmatrix}$	3. Transform to RREF (??) using elementary row operations.	• $(A^{-1})^{-1} = A$	products subtracted. This process is demonstrated below. $a = \begin{bmatrix} a_{11} & a_{12} \end{bmatrix}$
and rabils are present, the number of interactions is ∞ the product of the population sizes, which inverse behavior. Thus we can get the Lotas-Volterr Equations for the predictor prey model: $\begin{cases} \frac{dR}{dr} = a_R R - c_R R F \\ \frac{dR}{dr} = -a_R F - c_R R F \end{cases}$ (10) 9.9.2 Definitions	 a 8.2 Addition and Multiplication b Each new element in the matrix is a result of the dot product between the corresponding row and column matrices. 4 • A^T = A 	a new matrix. $[\mathbf{A}[\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ A_{21} & A_{22} & \cdots & A_{2m} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & b_n \end{bmatrix}$ (14)	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much easier to solve. 510.2.2 Prominent Vector Function Spaces 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trivial subspaces, calling the ⁶ 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} (16)$ $ A = a_{21} \cdot a_{11} - a_{22} \cdot a_{23}$ 11.2 Spanning Sets in \mathbb{R}^n
and rabils are present, the number of interactions is \propto the product of the population sizes, with inverse behavior. Thus we can get the Loda-Volter Equations for the prediator per ymodel: $\begin{cases} \frac{dR}{dr} = a_R R - c_R R F \\ \frac{dR}{dr} = -a_P F - c_F R F \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor.	 a 8.2 Addition and Multiplication Bach new element in the matrix is a result of the dot product between the corresponding row and column matrices. 4 • A² = A • If A ≠ 0, then A⁻¹ = ¹/_A. 	a new matrix. $[\mathbf{A}[\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ A_{21} & A_{22} & \cdots & A_{2m} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & b_n \end{bmatrix}$ (14)	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much easier to solve. 510.2.2 Prominent Vector Function Spaces ℝ² → The space of all ordered pairs. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the sero and the set V hemselves trial subspaces, calling the fore space of lines passing through the origin the only non-trivial subspace in ℝ². 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{12} \cdot a_{22}$ 11.2 Spanning Sets in \mathbb{R}^n A vector \overline{b} in \mathbb{R}^n is in Span $(\overline{y}, \overline{y},, \overline{y}_n)$ where $\{\overline{y}, \overline{y},, \overline{y}_n\}$ are vectors in \mathbb{R}^n votors in the state set one solution of the matrix- vectors in \mathbb{R}^n votors of the state set one solution of the matrix-
and rabils are present, the number of interactions is \propto the product of the population sizes, with inverse behavior. Thus we can get the Loda-Volter Equations for the prediator per ymodel: $\begin{cases} \frac{dR}{dr} = a_R R - c_R R F \\ \frac{dR}{dr} = -a_P F - c_F R F \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor.	 a 8.2 Addition and Multiplication b Each new element in the matrix is a result of the dot product between the corresponding row and column matrices. 4 • A^T = A 	a new matrix. $[A \mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ A_{21} & A_{22} & \cdots & A_{2m} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & b_n \end{bmatrix}$ (14)	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 510.2.2 Prominent Vector Function Spaces R² → The space of all ordered pairs. R² → The space of all ordered triples. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set Y themselves trivial subspaces, calling the 0 might be origin the origin the only non-trivial subspace in \$\$\$²\$ We can classify \$\$\$²\$ similarly: 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_2 \\ a_n & a_2 \end{bmatrix} (16)$ $ A = a_2 \cdot a_1 - a_n \cdot a_n$ 11.2 Spanning Sets in \mathbb{R}^n A vector 5 in \mathbb{R}^n is in Span($\mathbf{x}, \mathbf{x}, \dots, \mathbf{y}_n$) where $\{\mathbf{y}, \mathbf{x}, \dots, \mathbf{y}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vector equation $\mathbf{A}_n = \mathbf{b}$, where A is the matrix showed column vectors are
and rabils are present, the number of interactions is ∞ the product of M population airs, w this inverse behavior. Thus we can get the Lotala-Volter Equations for the predictor per model: $\begin{cases} \frac{dH}{dE} = a_{B}R - c_{B}RF \\ \frac{dH}{dE} = -a_{F}F - c_{F}RF \end{cases}$ (10) 9.0.2 Definitions Every element of $a \rightarrow v$ matrix has an associated minor and cofactor. • Minor $\rightarrow A (n - 1) \times (n - 1)$ matrix obtained by deleting the the rot	 a 8.2 Addition and Multiplication Each new element in the matrix is a result of the dat product between the corresponding row and column matrices. A^T = A If A ≠ 0, then A⁻¹ = ¹/_A; If (A) ≠ 0, an upper or lower triangle matrix², then the determinant is the 	a new matrix. $[\mathbf{A}[\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} $ (14) 1. $\vec{\mathbf{x}} + \vec{y} \in \mathcal{V}$ 2. $c\vec{\mathbf{x}} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{\mathbf{x}} + d\vec{y} \in \mathcal{V}$ which is called dosure under linear combinations.	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 510.2.2 Prominent Vector Punction Spaces ℝ² → The space of all ordered pairs. ℝ³ → The space of all ordered triples. ℝ^a → The space of all ordered triples. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trivial subspaces, calling the 6 subspace of lines passing through the origin the only non-trivial subspace in ℝ². We can classify ℝ³ similarly: Trivial: 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{12} \cdot a_{23}$ (16) 11.2 Spanning Sets in \mathbb{R}^n A vector \mathbf{F} in \mathbb{R}^n ; is in Span $(\mathbf{v}, \mathbf{v}, \dots, \mathbf{v}_n)$ where $\{\vec{\mathbf{v}}, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix-vector equation $A \mathbf{x} = \mathbf{\bar{k}}$, where A is the matrix whose column vectors are $\{\vec{\mathbf{v}}, \mathbf{v}_2, \dots, \mathbf{v}_n\}$.
and rabils are present, the number of interactions is \propto the product of the population sizes, with inverse behavior. Thus we can get the Lotla-Volderr Equations for the predictor prey model: $\begin{cases} \frac{dR}{dr} = a_R R - c_R R F \\ \frac{dR}{dr} = -a_R F - c_R R F \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow \lambda$ $(n - 1) \times (n - 1)$ matrix obtained by deleting the ih ro and jh column of A .	 a 8.2 Addition and Multiplication b) Each new element in the matrix is a result of the dot product between the corresponding true and column matrices. 4 • A^T = A • If (A ≠ 0, then A⁻¹ = ¹/_A; w • If A is an upper or lower triangle matrix³, then the determinant is the product of the diagonals. 	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & \mathbf{b} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & \mathbf{b} \end{bmatrix} $ (14) 1. $\mathbf{x} + \mathbf{y} \in \mathcal{V}$ 2. $\mathbf{c} \mathbf{x} \in \mathcal{V}$ which can be condensed into a single equation: $\mathbf{c} \mathbf{x} + d\mathbf{y} \in \mathcal{V}$ which is called closure under linear combinations.	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much easier to solve. 510.2.2 Prominent Vector Punction Spaces R² → The space of all ordered pairs. R³ → The space of all ordered netples. R³ → The space of all ordered netples. P → The space of all polynomials. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can all the zero and the set V themselves trivial subspaces, calling the 6 subspace of lines passing through the origin the only non-trivial subspace in R². We can calcassify R³ similarly: Trivial: Zero subspace 	products subtracted. This process is demonstrated below. $\begin{split} & A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} (16) \\ \hline & A = a_{22} \cdot a_{11} - a_{12} \cdot a_{23} \\ \hline & 11.2 \text{Spanning Sets in } \mathbb{R}^n \\ A vector & \text{In } \mathbb{R}^n \text{ in } [Span(q, q, \dots, q_n)] \text{ where } \{q, q_2, \dots, q_n\} \text{ are vectors in } \mathbb{R}^n, providel (\text{that there is at least one solution of the matrix-vector equation A^{g} = \mathbb{R}, where A is the matrix whose column vectors are \{q_1, q_2, \dots, q_n\}11.3 Span Theorem$
and rabils are present, the number of interactions is ∞ the product of M_{1} with inverse behavior. Thus we can get the Lotla-Volderr Equations for the predictor per y-model: $\begin{cases} \frac{dH}{dE} = a_{R}R - c_{R}RF \\ \frac{dH}{dE} = -a_{R}F - c_{R}RF \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow A (n - 1) \times (n - 1)$ matrix obtained by deleting the ith ro and j th column of A . • Cofactor \rightarrow The scalar $C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix A We can now determine a recursive method for any $n \times n$ matrix.	 a 8.2 Addition and Multiplication Bach new element in the matrix is a result of the dat product between the corresponding row and column matrices. 4 • A⁷ = A If (A) ≠ 0, then A⁻¹ = ¹/_A; w = A support or lower triangle matrix¹, then the determinant is the product of the diagonals. If one row or columns are equal, then A = 0. If we rows or columns are equal, then A = 0. A is invertible. 	a new matrix. $[\mathbf{A}[\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} $ (14) 1. $\vec{\mathbf{x}} + \vec{y} \in \mathcal{V}$ 2. $c\vec{\mathbf{x}} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{\mathbf{x}} + d\vec{y} \in \mathcal{V}$ which is called dosure under linear combinations.	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much easier to solve. 510.2.2 Prominent Vector Punction Spaces ℝ² → The space of all ordered pairs. ℝ³ → The space of all ordered n triples. ℙ → The space of all polynomials. ℙ_n → The space of all polynomials with degree ≤ n. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trivial subspaces, calling the fully subspace of lines passing through the origin the only non-trivial subspace in ℝ². We can classify ℝ³ similarly: Trivial: Zero subspace ℝ³ 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{12} \cdot a_{23}$ (16) 11.2 Spanning Sets in \mathbb{R}^n A vector \mathbf{F} in \mathbb{R}^n ; is in Span $(\mathbf{v}, \mathbf{v}, \dots, \mathbf{v}_n)$ where $\{\vec{\mathbf{v}}, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix-vector equation $A \mathbf{x} = \mathbf{\bar{k}}$, where A is the matrix whose column vectors are $\{\vec{\mathbf{v}}, \mathbf{v}_2, \dots, \mathbf{v}_n\}$.
and rabils are present, the number of interactions is ∞ the product of M population sizes, W in hirrswe behavior. Thus we can get the Lotas-Volter Equations for the predator per ymodel: $\begin{cases} \frac{4R}{2r} = a_R R - c_R RF \\ \frac{4R}{2r} = -a_F F - c_F RF \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow A(n - 1) \times (n - 1)$ matrix obtained by deleting the <i>i</i> th ro and <i>j</i> th column of <i>A</i> . • Cofactor \rightarrow The scalar $C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix <i>A</i>	 a 8.2 Addition and Multiplication b) Each new element in the matrix is a result of the dot product between the corresponding true and column matrices. 4 • A^T = A 4 (A^T = A 4 (A^T = A 4 (A^T = A 5 (A^T = A 6 (A is an upper or lower triangle matrix¹, then the determinant is the product of the diagonals. 6 In our row or column consists of only zeros, then A = 0. 16 If wo rows or columns are equal, then A = 0. 4 is a worklike. 	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & \mathbf{b} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & \mathbf{b} \end{bmatrix} $ (14) 1. $\mathbf{x} + \mathbf{y} \in \mathcal{V}$ 2. $\mathbf{c} \mathbf{x} \in \mathcal{V}$ which can be condensed into a single equation: $\mathbf{c} \mathbf{x} + d\mathbf{y} \in \mathcal{V}$ which is called closure under linear combinations.	 Transform to RREF (7?) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 510.2.2 Prominent Vector Function Spaces ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered rupples. ℙ → The space of all polynomials. ℝ₃ → The space of all polynomials with degree ≤ n. M_{men} → The space of all m × n matrices. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trial subspaces, calling the function of the origin the only non-trivial subspace in ℝ². We can classify ℝ³ similarly: Trivial: Zero subspace ℝ³ Non-Trivial 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{12} \cdot a_{23}$ (112) Spanning Sets in \mathbb{R}^n A vector 5 in \mathbb{R}^n is in Span $\{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n\}$ where $\{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vector equation $Ad = 5$, where A is the matrix whose column vectors are $\{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n\}$. 11.3 Span Theorem For a set of vectors $\{\mathbf{\tilde{v}}_1, \mathbf{\tilde{v}}_2, \dots, \mathbf{\tilde{v}}_n\}$ in vector space V, $\text{Span}(\mathbf{\tilde{v}}_1, \mathbf{\tilde{v}}_2, \dots, \mathbf{\tilde{v}}_n)$ is subspace of V.
and rabits are present, the number of interactions is ∞ the product of M_{1} Equations for the prediator pery model: $\begin{cases} \frac{M}{2} = a_{R} A - c_{R} R F \\ \frac{M}{2} = -a_{F} F - c_{F} R F \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow A (n - 1) \times (n - 1)$ matrix obtained by deleting the tilt ro and ph column of A . • Cofactor \rightarrow The scalar $C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix A We can now determine a recursive method for any $n \times n$ matrix. Using the definitions declared above, we use the recursive method the follows.	 a 8.2 Addition and Multiplication Back new element in the matrix is a result of the dat product between the corresponding row and column matrices. 4 . A^T = A If A ≠ 0, then A⁻¹ = ¹/_A; W if A is an upper or lower triangle matrix¹, then the determinant is the product of the diagonals. If one row or column consists of only zeros, then A = 0. If is also invertible. A his n pixel columns. 	a new matrix. $\begin{split} & [A \mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ A_{21} & A_{22} & \cdots & A_{2m} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & b_n \end{bmatrix} $ (14) 1. $\vec{\mathbf{x}} + \vec{\mathbf{y}} \in \mathcal{V}$ 2. $c\vec{\mathbf{x}} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{\mathbf{x}} + d\vec{\mathbf{y}} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones.	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much easier to solve. 510.2.2 Prominent Vector Punction Spaces ℝ² → The space of all ordered pairs. ℝ³ → The space of all ordered n triples. ℙ → The space of all polynomials. ℙ_n → The space of all polynomials with degree ≤ n. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trial subspace, calling the function of the origin the only non-trivial subspace in ℝ². We can classify ℝ³ similarly: Trivial: Zero subspace ℝ³ Non-Trivial Lines that contain the origin. Place that contain the origin. 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{11} \cdot a_{22} \end{bmatrix}$ (112 Spanning Sets in \mathbb{R}^{3} A vector $\overline{\mathbf{b}}$ in \mathbb{R}^{n} is in Span $\{\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n}\}$ where $\{\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n}\}$ are vectors in \mathbb{R}^{n} provided that there is at least one solution of the matrix- vectors in \mathbb{R}^{n} provided that there is at least one solution of the matrix- scale of $(\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n})$ in wetter space V, Span $\{\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n}\}$ is subspace of \mathbf{V} . 11.4 Column Space For any $n \times n$ matrix A , the column space, denoted Col A , is the span of
and rabils are present, the number of interactions is ∞ the product of M_{1} Equation for the prediator pery model: $\begin{cases} \frac{H}{2} = a_{0}R - c_{0}RF \\ \frac{H}{2} = -a_{0}F - c_{y}RF \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow A (n - 1) \times (n - 1)$ matrix obtained by deleting the three of an $d = 1 + (n - 1) + (n - 1)$ matrix obtained by deleting the three of $A = 0$. Cofactor $\rightarrow The$ scalar $C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix has the definitions declared by $R = 0$. Concrete $\rightarrow The$ scalar $C_{ij} = (C - 1)^{i+j} M_{ij}$ 9.9.3 Recursive Method of an $n \times n$ matrix has the definitions declared above, we use the recursive method the follows. $A = \sum_{j=1}^{n} a_{ij}C_{ij}$ (17)	 a 8.2 Addition and Multiplication Each new element in the matrix is a result of the dat predact between the corresponding row and column matrices. 4 . A^T = A If A ≠ 0, then A⁻¹ = ¹/_A; W If A is an upper or lower triangle matrix¹, then the determinant is the product of the diagonals. If one row or column consists of only zeros, then A = 0. If is also invertible. A has n pivot columns. (A) A = 0 	a new matrix. $\begin{bmatrix} [\mathbf{A}]\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{lm} & b_1 \\ A_{21} & A_{22} & \cdots & A_{lm} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{mm} & b_m \end{bmatrix} $ (14) 1. $\vec{\mathbf{x}} + \vec{\mathbf{y}} \in \mathcal{V}$ 2. $c\vec{\mathbf{x}} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{\mathbf{x}} + d\vec{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{\mathbf{x}} + \vec{\mathbf{y}} \in \mathcal{V} \leftarrow Addition$	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 510.2.2 Prominent Vector Function Spaces ℝ² → The space of all ordered pairs. ℝ³ → The space of all ordered rulps. ℝⁿ → The space of all ordered n-tuples. ℙ_n → The space of all polynomials. ℙ_m → The space of all polynomials with degree ≤ n. M_{mn} → The space of all ordered n-tuples. (Cl) → The space of all ordered n-tuples. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then no is A^T and (A⁻¹)^T = (A^T)⁻¹ If A is invertible, then no is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set Y theremetres trial subspace, calling the subspace of lines passing through the origin the only non-trivial subspace in ℝ¹. We can classify ℝ³ similarly: Trivial: Zero subspace ℝ³ Non-Trivial Lines that contain the origin. Places that contain the origin. 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{12} \cdot a_{23}$ 11.2 Spanning Sets in \mathbb{R}^n A vector \vec{b} in \mathbb{R}^n ; is in Span $(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n)$ where $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vector equation $A_n = \vec{b}$, where A is the matrix whose column vectors are $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$. 11.3 Span Theorem For a set of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ in vector space \mathbb{V} , $\text{Span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is subspace of \mathbb{V} . 11.4 Column Space
and rabits are present, the number of interactions is ∞ the product of M_{1} Equations for the prediator pery model: $\begin{cases} \frac{M}{2} = a_{R} A - c_{R} R F \\ \frac{M}{2} = -a_{F} F - c_{F} R F \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow A (n - 1) \times (n - 1)$ matrix obtained by deleting the tilt ro and ph column of A . • Cofactor \rightarrow The scalar $C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix A We can now determine a recursive method for any $n \times n$ matrix. Using the definitions declared above, we use the recursive method the follows.	 a 8.2 Addition and Multiplication Back new element in the matrix is a result of the dat product between the corresponding row and column matrices. A ≥ 0, then A⁻¹ = ¹/_A; If A ≥ 0, then A⁻¹ = ¹/_A; If a lay a supper or lower triangle matrix¹, then the determinant is the product of the diagonals. If one row or column consists of only zeros, then A = 0. If is non-invertible. A is invertible. A has n pivot columns. (A = 0) (A = 0) w fit A = 0 it is called singular, otherwise it is nonsingular. 	a new matrix. $\begin{bmatrix} [\mathbf{A}]\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ A_{21} & A_{22} & \cdots & A_{2m} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & b_n \end{bmatrix} $ (14) 1. $\vec{x} + \vec{y} \in \mathcal{V}$ 2. $c\vec{x} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{x} + d\vec{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{x} + \vec{y} \in \mathcal{V} \leftarrow \text{Addition}$ 2. $c\vec{x} \in \mathcal{V} \leftarrow \text{Scalar Multiplication}$	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 2.2 Prominent Vector Function Spaces ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered n-tuples. ℙ → The space of all polynomials. ℙ_n → The space of all polynomials. ℙ_n → The space of all polynomials. ℙ_n → The space of all polynomials. ∅_m → The space of all polynomials. ∅_m → The space of all polynomials. 	• $(A^{-1})^{-1} = A$ • A and B are invertible matrices of the same order if $(AB) = A^{-1}B^{-1}$ • If A is invertible, then so is A^{T} and $(A^{-1})^{T} = (A^{T})^{-1}$ We can call the zero and the set V themselves trivial subspaces, calling the δ subspace of lines passing through the origin the only non-trivial subspace in \mathbb{R}^{2} . We can calcassify \mathbb{R}^{3} similarly: • Trivial: - Zero subspace - \mathbb{R}^{3} Non-Trivial - Lines that contain the origin. 10.3.1 Examples	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{11} \cdot a_{22} \end{bmatrix}$ (112 Spanning Sets in \mathbb{R}^{3} A vector $\overline{\mathbf{b}}$ in \mathbb{R}^{n} is in Span $\{\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n}\}$ where $\{\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n}\}$ are vectors in \mathbb{R}^{n} provided that there is at least one solution of the matrix- vectors in \mathbb{R}^{n} provided that there is at least one solution of the matrix- scale of $(\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n})$ in wetter space V, Span $\{\vec{\mathbf{v}}_{1}, \vec{\mathbf{v}}_{2}, \dots, \vec{\mathbf{v}}_{n}\}$ is subspace of \mathbf{V} . 11.4 Column Space For any $n \times n$ matrix A , the column space, denoted Col A , is the span of
and rabils are present, the number of interactions is ∞ the product of M_{1} with inverse behavior. Thus we can get the Lotla-Adderr Equations for the predictor per ymodel: $\begin{cases} \frac{44}{45} = a_{1}R_{1} - c_{R}RF\\ \frac{44}{45} = -a_{1}F_{1} - c_{1}F_{1} - c_{1}F_{1}\\ \frac{44}{45} = -a_{1}F_{1} - c_{1}F_{1} - c_{1}F_{1}\\ \frac{44}{45} = -a_{1}F_{1} - a_{1}F_{2} - c_{1}F_{1} - c_{1}F_{1}\\ \frac{44}{45} = -a_{1}F_{1} - a_{1}F_{2} - c_{1}F_{1} - c_{1}F_{1}\\ \frac{44}{45} = -a_{1}F_{1} - a_{1}F_{2} - c_{1}F_{1} - c_{1}F_{1}\\ \frac{44}{45} = -a_{1}F_{1} - a_{1}F_{1} - a_{1}F_{1} - a_{1}F_{1}\\ \frac{44}{45} = -a_{1}F_{1} - a_{1}F_{1} -$	 a 8.2 Addition and Multiplication b) Each new element in the matrix is a result of the dot product between the corresponding true and column matrices. 4 • A^T = A • If (A) ≠ 0, then A⁻¹ = ¹/_A; w • If A is an upper or lower triangle matrix¹, then the determinant is the product of the diagonals. • If one row or column consists of only zeros, then A = 0. • If two rows or columna are equal, then A = 0. • A is mervitable. • A has n proto columns. () • A ≠ 0 • If A = 0 • If A = 0 	a new matrix. $\begin{bmatrix} [\mathbf{A}]\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ A_{21} & A_{22} & \cdots & A_{2m} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & b_n \end{bmatrix} $ (14) 1. $\vec{x} + \vec{y} \in \mathcal{V}$ 2. $c\vec{x} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{x} + d\vec{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{x} + \vec{y} \in \mathcal{V} \leftarrow \text{Addition}$ 2. $c\vec{x} \in \mathcal{V} \leftarrow \text{Scalar Multiplication}$ 3. $\vec{x} + \vec{\theta} = \vec{x} \leftarrow \text{Zero Element}$	 Transform to RREF (7) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 510.2.2 Prominent Vector Function Spaces ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered rupples. ℙ → The space of all ordered rupples. ℝ → The space of all polynomials. ℝ → The space of all polynomials with degree ≤ n. M_{mm} → The space of all continuous functions on the interval <i>I</i> (open, closel, finding and infinite). Cⁿ(<i>I</i>) → Sme as above, except with n continuous derivatives. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trivial subspaces, calling the 0 subspace of lines passing through the origin the only non-trivial subspace. Trivial Trivial: Zero subspace R³ Non-Trivial Lines that contain the origin. Places that contain the origin. 10.3.1 Examples The set of all even functions. The set of all solutions to g^H = 0 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{22} \end{bmatrix} (16)$ 14.1 2 Spanning Sets in \mathbb{R}^n A vector $\tilde{\mathbf{b}}$ in \mathbb{R}^n is Span $(\tilde{\mathbf{v}}, \tilde{\mathbf{v}}, \dots, \tilde{\mathbf{v}}_n)$ where $\{\tilde{\mathbf{v}}, \tilde{\mathbf{v}}_2, \dots, \tilde{\mathbf{v}}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vector equation $A_n = \tilde{\mathbf{v}}_n$ where $\{\tilde{\mathbf{v}}, \tilde{\mathbf{v}}_2, \dots, \tilde{\mathbf{v}}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vector equation $A_n = \tilde{\mathbf{v}}_n$ where $\{\tilde{\mathbf{v}}, \tilde{\mathbf{v}}_2, \dots, \tilde{\mathbf{v}}_n\}$ 11.3 Span Theorem For a set of vectors $\{\tilde{\mathbf{v}}, \tilde{\mathbf{v}}, \dots, \tilde{\mathbf{v}}_n\}$ in vector space \mathbb{V} , $Span\{\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2, \dots, \tilde{\mathbf{v}}_n\}is subspace of \mathbb{V}.11.4 Column SpaceFor any n \times n matrix A, the column space, denoted Col A, is the span ofthe column vectors of A, and is a subspace of [mathbhle?11.5 Linear IndependenceNew York in the calum vectors are call be reach by independentif no vector of the set can be vector the as a linear condition of the obsets.$
and rabits are present, the number of interactions is ∞ the product of M_{1} with inverse behavior. Thus we can get the Lotla-Adderr Equations for the predictor per model: $\begin{cases} \frac{44}{K} = a_{R}R - c_{R}RF\\ \frac{4}{K} = -a_{R}F - c_{R}RF \\ \frac{4}{K} = -a_{R}F - c_{R}RF \\ \frac{4}{K} = -a_{R}F - c_{R}RF \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow A$ ($n = 1$) $\times (n = 1$) matrix obtained by deleting the ih row and fh column of A . • Cofactor \rightarrow The scalar $C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix A We can now determine a rementive method for any $n \times n$ matrix. Tsing the definitions declared above, we use the recursive method the follows: $ A = \sum_{j=1}^{n} a_{ij}C_{ij} \qquad (17)$ Find j and then finish with the rules for the 2×2 matrix defined above in (77).	 a 8.2 Addition and Multiplication b) Each new element in the matrix is a result of the dot product between the corresponding true and column matrices. 4 • A^T = A • If A ≠ 0, then A⁻¹ = ¹/_A; w • If A is an upper or lower triangle matrix¹, then the determinant is the product of the diagonals. • If one row or column consists of only zeros, then A = 0. • If two rows or columna are equal, then A = 0. • A is meritable. • A is nevertable. • A has n pivot columns. (1) A ≠ 0 • If A = 0 it is called singular, otherwise it is nonsingular. 9.9.6 Cramer's Rule Far has n n matrix A with A ≠ 0, denote by A, the matrix obtained from A by replacing its the column vector. There then the column vector. 	a new matrix. $\begin{split} & \left \mathbf{A}[\mathbf{b}] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{2n} & b_1 \\ A_{11} & A_{22} & \cdots & A_{2n} & b_1 \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} & b_n \end{bmatrix} (14) \\ \hline & 1. \ \mathbf{x} + \mathbf{y} \in \mathcal{V} \\ 2. \ c\mathbf{x} \in \mathcal{Y} \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	 Transform to RREF (7) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to adve. 10.2.2 Prominent Vector Function Spaces R³ → The space of all ordered pairs. R³ → The space of all ordered numbers. P → The space of all ordered numbers. C ∩ → The space of all outmous functions on the interval I (open, closel, finite, and limiting). C'(I) → Sma as above, except with n continuous directives. C' → The space of all ordered numbers. 10.3 Vector Subspaces Theorem: A non-empty subset W of a vector space V is a subspace of V 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then no is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trivial subspace, calling the function of the subspace of lines passing through the origin the only non-trivial subspace in ℝ². We can classify ℝ³ similarly: Trivial: Zero subspace ℝ³ Non-Trivial Lines that contain the origin. Places that contain the origin. Diase that contain the origin. The set of all even functions. The set of all solutions to yⁿ − yⁿt + y = 0. 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_{11} & a_{22} \\ a_{21} & a_{22} \end{bmatrix} (16)$ (11) $ A = a_{22} \cdot a_{11} - a_{11} \cdot a_{22} \end{bmatrix}$ (16) (11) 11.2 Spanning Sets in \mathbb{R}^n A vector $\tilde{\mathbf{b}}$ in \mathbb{R}^n is in $\operatorname{Span}(\tilde{\mathbf{v}}_1, \tilde{\mathbf{v}}_2,, \tilde{\mathbf{v}}_n)$ where $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2,, \vec{\mathbf{v}}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- for $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2,, \vec{\mathbf{v}}_n\}$ are vectors are $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2,, \vec{\mathbf{v}}_n\}$ are the matrix- for a set of vectors $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2,, \vec{\mathbf{v}}_n\}$ in vector space V, $\operatorname{Span}(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2,, \vec{\mathbf{v}}_n)$ is subspace of V. 11.4 Column Space For any $n \times n$ matrix A , the column space, denoted Col A , is the span of the column vectors of A , and is a subspace of $ mathbdPe$. 13.5 Lincar Independence 14.5 Lincar Independence 15.6 Lincar of the equation to vector space \mathbb{V} is linearly independent. If no vector of the set can be written as a linear combination of the others.
and rabits are present, the number of interactions is ∞ the product of M_{i} with inverse behavior. Thus we can get the Lotla-Adderr Equations for the prediator pery model: $\begin{cases} \frac{qg}{dit} = a_{qR}R - c_{R}RF \\ \frac{qg}{dit} = a_{qR}F - c_{R}RF \\ \frac{qg}{dit} = a_{qR}F - c_{R}RF \\ \frac{qg}{dit} = a_{qR}F - c_{R}RF \end{cases} (10)$ 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • Minor $\rightarrow A$ ($n \rightarrow 1$) $\times (n \rightarrow 1)$ matrix obtained by deleting the thr va and jth column of A . • Cofactor \rightarrow The scalar $C_{ij} = (C - 1)^{i,j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix A We can now determine a recursive method for any $n \times n$ matrix. Using the definitions declared above, we use the recursive method the follows. $ A = \sum_{j=1}^{n} a_{ij}C_{ij}$ (17) Find j and then finish with the rules for the 2×2 matrix defined abov in (77). 9.9.4 Row Operations and Determinants Let A be square. • If two rows of A are exchanged to get B , then $ B = - A $.	 a 8.2 Addition and Multiplication Bach new element in the matrix is a result of the dat product between the corresponding tree and column matrices. 4 • A² = A If (A ≠ 0, then A⁻¹ = ¹/_A; if (A ≠ 0, then (A⁻¹) = ¹/_A; if the ange or clower triangle matrix¹, then the determinant is the product of the diagonals. if one row or column consists of only zeros, then A = 0. if two rows or columns are equal, then A = 0. if is also invertible. a. A is neitrible. b. As an pivot columns. if A = 0 if A = 0 if A = 0 Solution: if A = 0 if A = 0 if A = 0 if A = 0 9.4 A = 0 if A = 0 	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} & b_1 \\ \vdots & \vdots & \ddots & \vdots & b_n \end{bmatrix} (14)$ 1. $\mathbf{x} + \mathbf{y} \in \mathcal{V}$ 2. $\mathbf{c}^* \in \mathcal{V}$ which can be condensed into a single equation: $\mathbf{c}^* + d\mathbf{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\mathbf{x}^* + \mathbf{y} \in \mathcal{V} \leftarrow \text{Mittion}$ 2. $\mathbf{c}^* \in \mathcal{V} \leftarrow \text{Scalar Multiplication}$ 3. $\mathbf{x}^* + 0 = \mathbf{x} - \text{Zero Element}$ 4. $\mathbf{x} + (-\mathbf{x}) = (-\mathbf{x}) + \mathbf{x} = 0 \leftarrow \text{Additive Inverse}$ 5. $(\mathbf{x}^* + \mathbf{y}) + \mathbf{x} = \mathbf{x} + (\mathbf{y}^* + \mathbf{x}) \leftarrow \text{Associative Property}$	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 10.2.2 Prominent Vector Punction Spaces R² → The space of all ordered pairs. R³ → The space of all ordered pairs. R³ → The space of all ordered ruples. P_n → The space of all ordered ruples. P_n → The space of all ordered ruples. M_m → The space of all polynomials with degree ≤ n. M_m → The space of all continuous functions on the interval I (open, closed, finite, and infinite). Cⁿ(I) → Sine as above, except with n continuous derivatives. Cⁿ → The space of all ordered n-tuples of complex numbers. 3.3 Vector Subspaces 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then as is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the areo and the set Y themselves trial subspace. calling the function is R². We can classify R² similarly: Trivial: Zero subspace R² Non-Trivial Insection the origin. Places that contain the origin. Places that contain the origin. 10.3.1 Examples The set of all solutions to y^m - yⁿt + y = 0. (P ∈ P, P(2) = P(3)) 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{22} \end{bmatrix} (16)$ $ A = a_{22} \cdot a_{11} - a_{12} \cdot a_{23}$ (16) 11.2 Spanning Sets in \mathbb{R}^n A vector \mathbf{b} in \mathbb{R}^n is in $\operatorname{Span}(\mathbf{v}_1, \mathbf{v}_2,, \mathbf{v}_n)$ where $\{\mathbf{v}_1, \mathbf{v}_2,, \mathbf{v}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , given \mathbf{k}_1 and \mathbf{k}_2 , $, \mathbf{v}_n$ have $\{\mathbf{v}_1, \mathbf{v}_2,, \mathbf{v}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors are $\{\mathbf{v}_1, \mathbf{v}_2,, \mathbf{v}_n\}$ in vector space V, $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2,, \mathbf{v}_n\}$ is subspace of V. 11.4 Column Space For any $n \times n$ matrix A , the column space, denoted Col A , is the span of the column vectors $(A, \text{ and is a subspace of mathbble*.}$ 11.5 Linear Independence This notion of linear independence also carries over to function spaces. As et $\{\mathbf{v}_i, \mathbf{v}_{i-1}, \mathbf{v}_{i-1}\}$ of vectors in vectors are over to function spaces.
and rabils are present, the number of interactions is ∞ the product of M_{1} with inverse behavior. Thus we can get the Loda-Avderr Equations for the prediator prey model: $\begin{cases} \frac{H}{2} = a_{\mu}R - c_{\mu}RF \\ \frac{H}{2} = -a_{\mu}F - a_{\mu}F \\ \frac{H}{2} = -a_{\mu}F - a_{\mu}F \\ \frac{H}{2} = -a_{\mu}F - a_{\mu}F \\ \frac{H}{2} = -a_{\mu}F \\ \frac{H}{2} =$	 a 8.2 Addition and Multiplication Bach new element in the matrix is a result of the dat product between the corresponding tree and column matrices. 4 • A² = A If (A ≠ 0, then A⁻¹ = ¹/_A; if (A ≠ 0, then (A⁻¹) = ¹/_A; if the ange or clower triangle matrix¹, then the determinant is the product of the diagonals. if one row or column consists of only zeros, then A = 0. if two rows or columns are equal, then A = 0. if is also invertible. a. A is neitrible. b. As an pivot columns. if A = 0 if A = 0 if A = 0 Solution: if A = 0 if A = 0 if A = 0 if A = 0 9.4 A = 0 if A = 0 	a new matrix. $\begin{split} \ [\mathbf{A}[\mathbf{b}] \ &= \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} & b_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} & b_n \end{bmatrix} \tag{14}$ 1. $\vec{\mathbf{x}} + \vec{\mathbf{y}} \in \mathcal{V}$ 2. $(\vec{\mathbf{x}} \in \mathcal{V})$ which can be condensed into a single equation: $c\vec{\mathbf{x}} + d\vec{\mathbf{y}} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{\mathbf{x}} + \vec{\mathbf{y}} \in \mathcal{V} - \text{Addition}$ 2. $c\vec{\mathbf{x}} \in \mathcal{V} + \text{Solat Multiplication}$ 3. $\vec{\mathbf{x}} + \vec{0} = \vec{\mathbf{x}} - \text{Zero Element}$ 4. $\vec{\mathbf{x}} + (-\vec{\mathbf{x}}) = (-\vec{\mathbf{x}}) + \vec{\mathbf{x}} = \vec{0} + \text{Additive Inverse}$ 5. $(\vec{\mathbf{x}} + \vec{\mathbf{y}} = \vec{\mathbf{x}} + \vec{\mathbf{x}} + (\vec{\mathbf{y}} = \vec{\mathbf{z}} + \vec{\mathbf{y}} \neq \vec{\mathbf{x}} \leftarrow \text{Commutativity}$	 Transform to RREF (??) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 20.2.2 Prominent Vector Punction Spaces R² → The space of all ordered pairs. R³ → The space of all ordered pairs. R³ → The space of all ordered n-tuples. P_n → The space of all ordered n-tuples. P_n → The space of all ordered n-tuples. M_m → The space of all ordered n-tuples. M_m → The space of all nordered n-tuples. C₁(1) → The space of all nordered n-tuples. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. C'(1) → Sime as above, except with n continuous diructives. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then as is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the areo and the set Y themselves trial subspace. calling the function is R². We can classify R² similarly: Trivial: Zero subspace R² Non-Trivial Insection the origin. Places that contain the origin. Places that contain the origin. 10.3.1 Examples The set of all solutions to y^m - yⁿt + y = 0. (P ∈ P, P(2) = P(3)) 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{22} \end{bmatrix} $ (16) 111.2 Spanning Sets in \mathbb{R}^n A vector \mathbf{b} in $\mathbb{S}pan(\mathbf{v}_1, \mathbf{v}_{2,,\mathbf{v}_n})$ where $\{\mathbf{v}_1, \mathbf{v}_{2,,\mathbf{v}_n}\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vector equation $A(\mathbf{v}_1, \mathbf{v}_{2,,\mathbf{v}_n})$ where $\{\mathbf{v}_1, \mathbf{v}_{2,,\mathbf{v}_n}\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- ber as et of vectors $\{\tilde{v}_1, \tilde{v}_{2,,\mathbf{v}_n}\}$ in vector space V, $\mathrm{Span}(\tilde{v}_1, \tilde{v}_2,, \tilde{v}_n)$ is subspace of V. 11.4 Column Space For any $n \times n$ mutrix A , the column space, denoted Col A , is the span of the column vectors of A , and is a subspace of <i>probability</i> . 11.5 Linear Independente . Otherwise it is linearly dependent. Otherwise it is linearly dependent. Otherwise it is linearly dependent. More of vector for the fore $\{n_2, \dots, n_n\}$ in a vector space V is linearly independent of an interval I if or all i . It he only solution of \mathbf{e}^n , $\mathbf{e}_{n,2}$ in $\mathbf{e}_n \in \mathbf{e}$ and \mathbf{v} is the ord all i .
and rabils are present, the number of interactions is ∞ the product of M_{1} with inverse behavior. Thus we can get the Lotla-Avdohrr Equations for the prediator pery model: $\begin{cases} \frac{H}{M} = a_{II}R - c_{II}RF \\ \frac{H}{M} = -a_{II}F - c_{II}F \\ \frac{H}{M} = -a_{II}F - a_{II}F \\ \frac{H}{M} = -a_{II}F - a_{II}F \\ \frac{H}{M} = -a_{II}F \\ \frac{H}{M} = -$	 a 8.2 Addition and Multiplication Bach new element in the matrix is a result of the dat product between the corresponding tree and column matrices. 4 • A² = A If (A ≠ 0, then A⁻¹ = ¹/_A; if (A ≠ 0, then (A⁻¹) = ¹/_A; if the ange or clower triangle matrix¹, then the determinant is the product of the diagonals. if one row or column consists of only zeros, then A = 0. if two rows or columns are equal, then A = 0. if is also invertible. a. A is neitrible. b. As an pivot columns. if A = 0 if A = 0 if A = 0 Solution: if A = 0 if A = 0 if A = 0 if A = 0 9.4 A = 0 if A = 0 	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} & b_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{41} & A_{42} & \cdots & A_{4m} & b_n \end{bmatrix} $ (14) 1. $\vec{x} + \vec{y} \in \mathcal{V}$ 2. $c\vec{x} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{x} + d\vec{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{x} + \vec{y} \in \mathcal{V} \leftarrow \text{Addition}$ 2. $c\vec{x} \in \mathcal{V} \leftarrow \text{Solar Multiplication}$ 3. $\vec{x} + \vec{0} = \vec{x} \leftarrow \text{Zero Element}$ 4. $\vec{x} + (-\vec{x}) = (-\vec{x}) + \vec{x} = \vec{0} \leftarrow \text{Additive Inverse}$ 5. $(\vec{x} + \vec{y}) = \vec{y} + \vec{x} - \vec{C} \text{commattivity}$ 6. $\vec{x} + \vec{y} = \vec{y} + \vec{x} \leftarrow \text{Commattivity}$ 7. $1 \cdot \vec{x} = \vec{x} \leftarrow \text{Identity}$	 Transform to RREF (?!) using dementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much cause to solve. 10.2.2 Prominent Vector Function Spaces ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered pairs. ℝ³ → The space of all ordered n-tuples. ℝ⁴ → The space of all ordered n-tuples. Ω₁ → The space of all n ordered n-tuples. C(I) → The space of all ordered n-tuples of complex numbers. C(I) → The space of all ordered n-tuples of complex numbers. C⁴ → The space of all ordered n-tuples of complex numbers. C⁴ → The space of all ordered n-tuples of complex numbers. I f u ≤ 4 c, W, than d f = 0 set of a sector space V is a subspace of V if it is closed under addition and seclar multiplication: If u ≤ 4 c, W, than d f = \$\vec{V}\$. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then no is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trivial subspace. Calling the function of the set of	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_3 \end{bmatrix} \qquad (16)$ (112) Spanning Sets in \mathbb{R}^n A vector \mathbf{b} in $\mathbf{k} = \mathbf{k}_2 \cdot \mathbf{e}_1 - \mathbf{e}_3 \cdot \mathbf{e}_3$ (16) 11.2 Spanning Sets in \mathbb{R}^n A vector \mathbf{b} in \mathbb{R}^n as a last one solution of the matrix-vector equation $A\mathbf{k} = \mathbf{b}$, where A is the matrix whose column vectors are $\{(v_1, v_2, \dots, v_d)\}$ are vectors in \mathbb{R}^n , provided that there is at last one solution of the matrix-vector equation $A\mathbf{k} = \mathbf{b}$, where A is the matrix whose column vectors are $\{(v_1, v_2, \dots, v_d)\}$ is subspace of V. 11.3 Span Theorem For a set of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k\}$ in vector space \mathbb{V} . Span $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k\}$ is subspace of V. 11.4 Column Space For any $n \times n$ matrix A , the column space, denoted Col A , is the span of the column vectors of A , and is a subspace of published ² . 11.5 Linear Independence A set $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k\}$ of vectors in vector space \mathbb{V} is linearly independent if no vector of these is unark vectors $(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k)$ in vector space. The solution of four independence. The solution of linear independence are interval M if for \mathbf{v} is the interval M is the vector space \mathbb{V} is linearly independent indiversed in $(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k)$ in the vector space \mathbb{V} is linearly independent and interval M if \mathbf{v} is the interval M is the vector space \mathbb{V} is linearly independent and interval M if \mathbf{v} is the vector space \mathbb{V} is linearly independent on a linear \mathbb{V} if M is M in $(\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_2)$ is independent on $(\mathbf{v}_1 + \mathbf{v}_2)$ is a vector space \mathbb{V} is linearly independent.
and rabils are present, the number of interactions is ∞ the product of M_{1} with inverse behavior. Thus we can get the Loda-Avderr Equations for the prediator prey model: $\begin{cases} \frac{H}{2} = a_{\mu}R - c_{\mu}RF \\ \frac{H}{2} = -a_{\mu}F - a_{\mu}F \\ \frac{H}{2} = -a_{\mu}F - a_{\mu}F \\ \frac{H}{2} = -a_{\mu}F - a_{\mu}F \\ \frac{H}{2} = -a_{\mu}F \\ \frac{H}{2} =$	 a. 8.2 Addition and Multiplication b) Each new element in the matrix is a result of the dat predoct between the corresponding row and column matrices. 4 . A^T = A if A ≠ 0, then A⁻¹ = ¹/_A!; if A ≠ 0, then A⁻¹ = ¹/_A!; if one row or column consists of only zeros, then A = 0. if is no invertible. A is invertible. A is invertible. if is also invertible. if A = 0 the scalar distribution of the system is it is nonsingular. Def the new or columns. if A = 0 if A = 0 is called singular, otherwise it is nonsingular. Bor the n × n matrix A with A ≠ 0, denote by A, the matrix column from Ab replacing its its hooting in given by: \$	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nm} & b_n \end{bmatrix} $ (14) 1. $\vec{\mathbf{x}} + \vec{\mathbf{y}} \in \mathcal{V}$ 2. $(\vec{\mathbf{x}} \in \mathcal{V})$ which can be condensed into a single equation: $c\vec{\mathbf{x}} + d\vec{\mathbf{y}} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{\mathbf{x}} + \vec{\mathbf{y}} \in \mathcal{V} \leftarrow Addition$ 2. $(\vec{\mathbf{x}} \in \mathcal{V} \leftarrow Cadit Multiplication$ 3. $\vec{\mathbf{x}} + \vec{0} = \vec{\mathbf{x}} \leftarrow Caro Element$ 4. $\vec{\mathbf{x}} + (-\vec{\mathbf{x}}) = (-\vec{\mathbf{x}}) + \vec{\mathbf{x}} = \vec{0} \leftarrow Additive Inverse$ 5. $(\vec{\mathbf{x}} + \vec{\mathbf{y}}) = \vec{\mathbf{x}} = \vec{\mathbf{x}} + (\vec{\mathbf{y}} + \vec{\mathbf{x}}) \leftarrow Amountativity$ 7. $1 \cdot \vec{\mathbf{x}} = \vec{\mathbf{x}} \leftarrow Identity$ 8. $c(\vec{\mathbf{x}} + \mathbf{y}) = c\vec{\mathbf{x}} + c\vec{\mathbf{y}} \leftarrow Dastributive Property$	 3. Transform to RREF (?f) using elementary row operations. 4. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to adve. 510.2.2. Frominent Vector Function Spaces R² → The space of all ordered pairs. R² → The space of all ordered pairs. R² → The space of all ordered rupples. R² → The space of all ordered rupples. R² → The space of all ordered noise. R² → The space of all ordered noise. R³ → The space of all ordered noise. R³ → The space of all ordered noise. R⁴ → The space of all ordered noise. M⁴ → The space of all ordered noise. M⁴ → The space of all ordered noises functions on the interval I (open, closed, finite, and infinite). C⁴(I) → Sma ea above, except with n continuous derivatives. C⁴ → The space of all ordered n-tuples of complex numbers. 10.3 Vector Subspaces Theorem: A nonempty subset W of a vector space V is a subspace of V if it is closed under addition and scalar multiplication: If d(q < W, than d + \Vec W. If all < W and c < B, than cit W. 	$\begin{array}{l} \cdot (A^{-1})^{-1} = A\\ \cdot A \mbox{ and } B \mbox{ are invertible matrices of the same order if (AB) = A^{-1}B^{-1}\\ \cdot \mbox{ if } A \mbox{ is invertible, then as is } A^{T} \mbox{ and } (A^{-1})^{T} = (A^{T})^{-1}\\ \hline \mbox{ We can call the zero and the set } V \mbox{ theoregin the origin the only non-trivial subspaces, calling the 0 migrit \mathbb{R}^{2} is millarly:\cdot \mbox{ Trivial}- \mbox{ Zero subspace} \\ - \mathbb{R}^{3} \mbox{ mom} \mbox{ the origin.} \\ - \mbox{ Places that contain the origin.} \\ - \mbox{ Places that contain the origin.} \\ \mbox{ 10.3.1 Examples} \mbox{ of all even functions.} \\ \cdot \mbox{ The set of all solutions to } y^{\prime\prime} - y^{\prime}t + y = 0. \\ \cdot \box{ (} P \in \mathbb{P}, P(2) = P(3)\box{)} \\ \mbox{ 11 Span, Basis and Dimension} \\ \mbox{ 11.1 Span} \\ \mbox{ The set of al (\psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the origin of the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the origin of the origin of the origin of the origin vectors in a vector space V, denoted by the origin of the $	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_{22} \\ a_{31} & a_{32} \end{bmatrix} $ (16) 11.1 Spanning Sets in \mathbb{R}^n A vector \mathbf{f} in \mathbb{R}^n is in Span $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_{n-1}, -\vec{\mathbf{v}}_{n-1} \neq a_{2n} \neq $
and rabbins are present, the number of interactions is ∞ the product of M propulation along, with inverse behavior. Thus we can get the Lodas Avolter Equations for the prediator prey model: $\begin{cases} \frac{M}{2} = a_R R - c_R R F \\ \frac{M}{2} = -a_F F - c_C R F \end{cases} $ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. • $Mmo \to A$ $(n-1) \times (n-1)$ matrix obtained by deleting the thr or and fh column of A . • $Cofactor \to The standard G_1 = (C - 1)^{1/2} M_{01} 9.9.3 Recursive Method of an n \times n matrix AWe can now determine a recursive method for any n \times n matrix.Using the definitions declared above, we use the recursive method the follows. A = \sum_{j=1}^{n} n_j G_{ij} (12)Find j and then finish with the rules for the 2 \times 2 matrix defined above in (72).9.9.4 Row Operations and DeterminantsLet A be square.I if two rows of A is sumhjpiled by a constant c_i and then added to another row to get B, then B = - A .• If one row of A is sumhjpiled by a constant c_i and then B = c A .$	 a. 8.2 Addition and Multiplication Bech new element in the matrix is a result of the dat product between the corresponding row and column matrices. 4 • A^T = A If (A ≠ 0, then A⁻¹ = ¹/_A). If (A ± 0, then A⁻¹ = ¹/_A). If or more or column consists of only zeros, then A = 0. If two rows or columns are equal, then A = 0. If two rows or column consists of only zeros, then A = 0. If we rows or column consists of only zeros, then A = 0. If we rows or column consists of only zeros, then A = 0. If we rows or column consists of only zeros, then A = 0. If we rows or column consists of only zeros, then A = 0. If we rows or column consists of only zeros, then A = 0. If a lo is invertible. If A = 0 is called singular, otherwise it is nonsingular. 9.0.6 Carmer's Rule For the solver Rule For the solver rows are the old the column vector b. Then the sht component of the solution of the system is given by:	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} & b_1 \\ \vdots & \vdots & \ddots & \vdots \\ A_{41} & A_{22} & \cdots & A_{2m} & b_2 \\ \vdots & \vdots & \ddots & \vdots \\ A_{41} & A_{42} & \cdots & A_{4m} & b_1 \end{bmatrix} $ ((4) 1. $\vec{x} + \vec{y} \in \mathcal{V}$ 2. $c\vec{x} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{x} + d\vec{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{x} + \vec{y} \in \mathcal{V} \leftarrow Addition$ 2. $c\vec{x} \in \mathcal{V} \leftarrow Salar Multiplication$ 3. $\vec{x} + \vec{0} = \vec{x} \leftarrow Zoro Element$ 4. $\vec{x} + (-\vec{x}) = (-\vec{x}) + \vec{x} = \vec{0} \leftarrow Additive Inverse$ 5. $(\vec{x} + \vec{y}) + \vec{x} = -\vec{0} \leftarrow Additive Inverse$ 5. $(\vec{x} + \vec{y}) = \vec{y} + \vec{x} - Commutativity$ 7. $1 \cdot \vec{x} = \vec{x} \leftarrow Identity$ 8. $c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y} \leftarrow Distributive Property$ 9. $(c + d)\vec{x} = c\vec{x} + d\vec{k} \leftarrow Distributive Property$ 10. $c(d\vec{x}) = (cd)\vec{y} \leftarrow Associativity$	 Transform to RREF (?f) using elementary row operations. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 10.2.2. Prominent Vector Function Spaces R² → The space of all ordered pairs. R³ → The space of all ordered pairs. R³ → The space of all ordered pairs. R³ → The space of all ordered rulples. P → The space of all ordered n-tuples. P → The space of all polynomials. P → The space of all polynomials. P → The space of all polynomials. Q(I) → The space of all polynomials. C(I) → The space of all n ×n matrices. C(I) → The space of all nontimous functions on the interval I (open, closed, finite, and infinite). C(I) → The space of all continuous functions on the interval I (open, closed, finite, and infinite). C(I) → The space of all continuous functions on the interval I (open, closed, finite, and infinite). C(I) → The space of all ordered n-tuples of complex numbers. C → The space of all ordered n-tuples of complex numbers. Hortorn: A non-empty solvest W of a vector space V is a subspace of V if it is closed under addition and scalar multiplication: If d q \in W, that at ¹ \not W. If d \not W and c \not R, that at \not \not W. If u \not q \not R at a \not \not W. If u \not q \not R at \not \not W. If u \not q \not R at \not \not W. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then so is A^T and (A⁻¹)^T = (A^T)⁻¹ We can call the zero and the set V themselves trial subspace, calling the function of the analysis of the analysis of the origin the only non-trivial subspace of the set V themselves trial subspace. All the set on the set V themselves trial subspace of the set V themselves trial subspace. All the set of all even functions. The set of all even functions. (P ∈ F, P(2) = P(3)) 11 Span, Basis and Dimension 11.1 Span 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_{22} \\ a_{31} & a_{32} \end{bmatrix} $ (16) 11.1 Spanning Sets in \mathbb{R}^n A vector \mathbf{f} in \mathbb{R}^n is in Span $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_{n-1}, -\vec{\mathbf{v}}_{n-1} \neq a_{2n} \neq $
and rabils are present, the number of interactions is ∞ the product of M Equation for the prediator pery model: $\begin{cases} \frac{H}{2} = a_{\mu}R - c_{\mu}RF \\ \frac{H}{2} = -a_{\mu}F - c_{\mu}RF \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. (Minor $\rightarrow A (n - 1) \times (n - 1)$ matrix obtained by deleting the throw and phy column of A . (Minor $\rightarrow A (n - 1) \times (n - 1)$ matrix obtained by deleting the throw and phy column of A . (Construction $\rightarrow The$ scalar $C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.9.3 Recursive Method of an $n \times n$ matrix has Using the definitions declared above, we use the recursive method the follows. $ A = \sum_{j=1}^{n} a_{ij}C_{ij}$ (17) 9.9.4 Row Operations and Determinants Let A be square. I throw or A is multiplied by a constant c , and then added to another nor $oth A$ is multiplied by a constant c , then $ B = c A $. Is for one or A is multiplied by a constant c , then $ B = c A $.	 a 8.2 Addition and Multiplication Bech are element in the matrix is a result of the dat product between the corresponding row and column matrices. A⁻¹ = A If (A ≠ 0, then A⁻¹ = ¹/_A]. If (A ≠ 0, then A⁻¹ = ¹/_A]. If one ore or column consists of only zeros, then A = 0. If we rows or columns are set of µ = 0. If we rows or columns are equal, then A = 0. A is invertible. A⁻¹ is also invertible. A is invertible. A ≠ 0 A = 0 A = 0 Both are stables of the solution stables of the product between the the component of the solution of the system is given by: m x_i = ¹/_A (18) D O Vector Spaces and Subspaces A we can define the van define the operation of the system of the solution and the system of the solution and the system of the solution and the system of the solution of of the solo	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} \\ = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{bn} \\ A_{11} & A_{22} & \cdots & A_{bn} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{bn} \end{bmatrix} \begin{bmatrix} b_{1} \\ b_{2} \end{bmatrix} $ (14) 1. $\mathbf{x} + \mathbf{y} \in \mathcal{V}$ 2. $\mathbf{x}' \in \mathcal{V}$ which can be condensed into a single equation: $\mathbf{x}' + d\mathbf{y}' \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\mathbf{x}' + \mathbf{y}' \in \mathcal{V} \leftarrow \text{Addition}$ 2. $\mathbf{x}' \in \mathcal{V} \leftarrow \text{Scalar Multiplication}$ 3. $\mathbf{x}' = 0 = \mathbf{x}' - \text{Zero Element}$ 4. $\mathbf{x}' + (-\mathbf{x}) = (-\mathbf{x}) + \mathbf{x} = 0 \leftarrow \text{Additive Inverse}$ 5. $(\mathbf{x}' + \mathbf{y}') = \mathbf{x}' + \mathbf{x}' \leftarrow \text{Diarmativity}$ 7. $1 \cdot \mathbf{x}' = \mathbf{x}' + \text{Identity}$ 8. $(\mathbf{x}' + \mathbf{y}) = \mathbf{x}' + \mathbf{x}' \leftarrow \text{Diartbuttive Property}$ 9. $(\mathbf{c} + d\mathbf{y} = \mathbf{x}' + d\mathbf{x} \leftarrow \text{Diartbuttive Property}$ 10. $\mathbf{c}d\mathbf{x}' = (\mathbf{a}')\mathbf{x} \leftarrow \text{Associativity}$ 10.2 Vector Function Space	 Transform to RREF (7) using elementary row operations. The linear matrix formed by this process has the same solutions as the finite system, however it is much casier to solve. 510.2.2 Prominent Vector Punction Spaces R³ → The space of all ordered pairs. R³ → The space of all ordered pairs. R³ → The space of all ordered rupples. R → The space of all ordered rupples. R → The space of all polynomials with degree ≤ n. M_{ma} → The space of all continuous functions on the interval I (open, closel, find, f	$\begin{array}{l} \cdot (A^{-1})^{-1} = A\\ \cdot A \mbox{ and } B \mbox{ are invertible matrices of the same order if (AB) = A^{-1}B^{-1}\\ \cdot \mbox{ if } A \mbox{ is invertible, then as is } A^{T} \mbox{ and } (A^{-1})^{T} = (A^{T})^{-1}\\ \hline \mbox{ We can call the zero and the set } V \mbox{ theoregin the origin the only non-trivial subspaces, calling the 0 migrit \mathbb{R}^{2} is millarly:\cdot \mbox{ Trivial}- \mbox{ Zero subspace} \\ - \mathbb{R}^{3} \mbox{ mom} \mbox{ the origin.} \\ - \mbox{ Places that contain the origin.} \\ - \mbox{ Places that contain the origin.} \\ \mbox{ 10.3.1 Examples} \mbox{ of all even functions.} \\ \cdot \mbox{ The set of all solutions to } y^{\prime\prime} - y^{\prime}t + y = 0. \\ \cdot \box{ (} P \in \mathbb{P}, P(2) = P(3)\box{)} \\ \mbox{ 11 Span, Basis and Dimension} \\ \mbox{ 11.1 Span} \\ \mbox{ The set of al (\psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the origin of the origin of the set (\psi_{n}, \psi_{n}, \dots, \psi_{n}) of vectors in a vector space V, denoted by the origin of the origin of the origin of the origin of the origin vectors in a vector space V, denoted by the origin of the $	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_3 \\ \alpha_4 \end{bmatrix} \begin{pmatrix} \alpha_1 & \alpha_5 \\ \alpha_{12} & \alpha_{12} \end{pmatrix} (16)$ 11.1 Spanning Sets in R³ A vector 5 in R ^a is in Span($\vec{q}_1, \vec{q}_2,, \vec{q}_n$) where { $\vec{q}_1, \vec{q}_2,, \vec{q}_n$ } are vector equilibrium provided that there is at least one solution of the matrix- vector equilibrium A^{-1} is the set is the set one solution of the matrix- vector equilibrium A^{-1} is where A is the matrix whose column vectors are { $\vec{q}_1, \vec{q}_2,, \vec{q}_n$ } is number of the matrix- for a set of vectors { $\vec{v}_1, \vec{v}_2,, \vec{v}_n$ } in vector space V, Span{ $\vec{v}_1, \vec{v}_2,, \vec{v}_n$ } is subspace of V. 11.4 Column Space For any $n \times n$ matrix A , the column space, denoted Col A , is the span of the column vectors of A , and is a subspace of <i>[mathbble</i>] ^c . 11.5 Lincar Independence 13.6 $\vec{v}_1 \vec{v}_2,, \vec{v}_n$ if no vector space V is linearly independent on an interval I if or all I in I the only solution of n^2 , r_2 as of $\vec{v}_1 \vec{v}_2,, \vec{v}_n$ if \vec{v}_1 is avector space V is functionally independent on an interval I if or all I in I the only solution \vec{v}_1, \vec{v}_2 . The totion of linear independence also cartes over to functiona spaces. A set of vector for \vec{b} ($\vec{v}_1,, \vec{v}_n$ if n avector \vec{a} is $\vec{v}_1 \vec{v}_2 \vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1$ is avector \vec{a} in $\vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1$ is avector \vec{a} in $\vec{v}_1 \vec{v}_2 \vec{v}_1$ is avector \vec{a} in $\vec{v}_1 \vec{v}_2 \vec{v}_1$ is avector \vec{a} in $\vec{v}_1 \vec{v}_1 \vec{v}_1$ in $\vec{v}_1 \vec{v}_1 \vec{v}_1$ is the vector \vec{a} in $\vec{v}_1 \vec{v}_1 \vec{v}_1$ is $\vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1$ in $\vec{v}_1 \vec{v}_1 \vec{v}_1$ is $\vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1$ in $\vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1$ is $\vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1$ in $\vec{v}_1 \vec{v}_1 \vec{v}_1$ is $\vec{v}_1 \vec{v}_1 \vec{v}_1 \vec{v}_1$ in $\vec{v}_1 \vec{v}_1 \vec{v}_$
and rabils are present, the number of interactions is ∞ the product of M Equation for the prediator pery model: $\begin{cases} \frac{H}{2} = a_{\mu}R - c_{\mu}RF \\ \frac{H}{2} = -a_{\nu}F - c_{\nu}RF \end{cases}$ (10) 9.9.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw and phy column of A . (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw and phy column of A . (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\rightarrow \Lambda (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\Lambda (n-1) \times (n-1) \times (n-1)$ matrix obtained by deleting the throw (Minor $\Lambda (n-1) \times $	* 8.2 Addition and Multiplication 1) Each new element in the matrix is a result of the dat product between the corresponding row and column matrices. 4 $\cdot A^{T} = A$ $\cdot f(A \neq 0, then A^{-1} = \frac{1}{\Delta t};* If A is an upper or lower triangle matrix1, then the determinant is theproduct of the diagonals.• If one row or column consists of only zeros, then A = 0.• If two rows or columns do only zeros, then A = 0.• If we rows or columns equal, then A = 0.• A is invertible.• A is invertible.• A is a not proto columns.• A \neq 0• If A = 0 it is called singular, otherwise it is nonsingular.9.0 Craner's RuleFor the n \times n matrix A with A \neq 0 denote by A, the matrix obtainedfrom A by replacing its it is column with the column vector b. Then the stricecomponent of the solution of the system is given by:• x = \frac{ A }{ A } (18)10 Vector Spaces and Subspaces$	a new matrix. $\begin{bmatrix} A b = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} & b_{1} \\ \vdots & \vdots & \ddots & \vdots & b_{2} \\ \vdots & \vdots & \ddots & \vdots & b_{2} \end{bmatrix} $ (14) 1. $\mathbf{x} + \mathbf{y} \in \mathcal{V}$ 2. $c\mathbf{x} \in \mathcal{V}$ which can be condensed into a single equation: $c\mathbf{x} + d\mathbf{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\mathbf{x} + \mathbf{y} \in \mathcal{V} - \text{Addition}$ 2. $c\mathbf{x} \in \mathcal{V}$ be Solar Multiplication 3. $\mathbf{x} + 0 = \mathbf{x} - \text{Zero Element}$ 4. $\mathbf{x} - (-\mathbf{x}) - (-\mathbf{x}) + \mathbf{x} = 0 \leftarrow \text{Additive Inverse}$ 5. $(\mathbf{x} + \mathbf{y}) = \mathbf{y} + \mathbf{x} - \text{Commutativity}$ 7. $1 \cdot \mathbf{x} = \mathbf{x} + \mathbf{y} + \mathbf{z} = \mathbf{x} + \mathbf{y} + \mathbf{z}) \leftarrow \text{Associative Property}$ 8. $(c\mathbf{x}' + \mathbf{y}) = c\mathbf{x} + c\mathbf{y} \leftarrow \text{Distributive Property}$ 9. $(c + d\mathbf{y}) = c\mathbf{x} + c\mathbf{y} \leftarrow \text{Distributive Property}$ 10. $c(d\mathbf{x}) = (ad)\mathbf{x} \leftarrow \text{Associativity}$ 10.2 Vector Function Space A vector function space is just a unique vector space where the elements of the space are functions.	 3. Transform to RREF (??) using elementary row operations. 4. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 50.2.2. Prominent Vector Function Spaces R³ → The space of all ordered pairs. R³ → The space of all ordered n-tuples. R → The space of all continuous functions on the interval I (open, identify the space of all ordered n-tuples of ordered n-tuples. C[*](I) → The space of all continuous functions on the interval I (open, identify the space of all ordered n-tuples of complex numbers. C[*](I) → Same as above, except with n continuous dirictives. C[*] → The space of all ordered n-tuples of complex numbers. DAS Vector Subspaces The development addition and scalar multiplication: If if q ∈ W, uhan if + V ∈ W. If if q ∈ W and a h ∈ R, than all + hθ ∈ W. (20) Nate, vector space does not imply subspace. All subspaces are vector spaces. The ordered not imply subspace. The determine if it is a subspace, we check for closure with the above theorem. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then no is A^T and (A⁻¹)^T = (A^T)⁻¹ We can called by the avera of the set V themselves trivial subspace. Calling the function of the set V themselves trivial subspace. Trivial: Zero subspace R³ Non-Trivial Lines that contain the origin. Places that contain the origin. Places that contain the origin. IDe set of all even functions. The set of all even functions. (P ∈ R; P(2) = P(3)) 11 Span, Basis and Dimension 11.4 Span U The set of (9, 9,, 4) of vectors in a vector space V, denoted by Span(§1, \$v_1,, \$v_1\$) is the set of all incer combinations of the vectors. 11.1 Example 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{22} \end{bmatrix} $ (16) $ A = a_{22} \cdot a_{11} - a_{12} \cdot a_{23}$ (17) 11.2 Spanning Sets in \mathbb{R}^n A vector \mathbf{b} in \mathbb{R}^n is in $\operatorname{Span}(\mathbf{v}_1, \mathbf{v}_2,, \mathbf{v}_n)$ where $\{\mathbf{v}_1, \mathbf{v}_2,, \mathbf{v}_n\}$ are vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is at least one solution of the matrix- vectors in \mathbb{R}^n , provided that there is a transformed by the set of the solution of the matrix- form \mathbf{s}^n , and \mathbf{s}^n , \mathbf{s}^n , \mathbf{s}^n , \mathbf{s}^n , \mathbf{s}^n is subspace of \mathbb{N} . 11.4 Column Space 11.5 Linear Independence This notion of linear independence also carries over to function spaces. As et $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2,, \vec{\mathbf{v}}_n\}$ of vectors in vectors in sector space \mathbb{V} is linearly independent in the oution of the independence also carries over to function spaces. As solver of the interiors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_1, \mathbf{v}_1$ and vector space \mathbb{V} is linearly independent This notion of linear independence also carries over to function spaces. As solver over functions $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_1, \mathbf{v}_1$ and \mathbf{v}_2 are \mathbb{V} is linearly independent functions \mathbb{R}^n independence \mathbb{R}^n is \mathbb{R}^n in \mathbb{R}^n in \mathbb{R}^n if \mathbb{R}^n if \mathbb{R}^n if \mathbb{R}^n is \mathbb{R}^n in \mathbb{R}^n is \mathbb{R}^n if \mathbb{R}^n if \mathbb{R}^n is \mathbb{R}^n is \mathbb{R}^n if \mathbb{R}^n if \mathbb{R}^n i
and rabils are present, the number of interactions is ∞ the product of M_{1} logarithm area, M_{1} winverse behavior. Thus we can get the Lotda-Adobter Equations for the prediator prey model: $\begin{cases} \frac{W}{2} = a_{\mu}F - c_{\mu}RF \\ \frac{W}{2} = -a_{\mu}F - c_{\mu}RF \end{cases}$ (10) 9.0.2 Definitions Every element of a $n \times n$ matrix has an associated minor and cofactor. (Minor $\rightarrow \Lambda (n - 1) \times (n - 1)$ matrix obtained by deleting the <i>i</i> th ro and <i>j</i> th column of <i>A</i> . • Cofactor $\rightarrow \text{The scalar } C_{ij} = (C - 1)^{i+j} M_{ij} $ 9.0.3.3 Recursive Method of an $n \times n$ matrix A We can now determine a recursive method for any $n \times n$ matrix. Diag the definitions declared above, we use the recursive method the follows. $ A = \sum_{n=1}^{\infty} a_{n}C_{ij} \qquad (17)$ 9.0.4 Row Operations and Determinants Let <i>A</i> be square. It if two rows of <i>A</i> are exchanged to get <i>B</i> , then $ B = - A $. • If one row <i>d A</i> is multiplied by a constant <i>c</i> , and then added to anothe row of <i>A B</i> is multiplied by a constant <i>c</i> , and hen added to another row to get <i>B</i> , then $ A = B $.	* 8.2 Addition and Multiplication 1) Each new element in the matrix is a result of the dat predoct between the corresponding row and column matrices. 4 . $ A ^2 = A$. If $ A \neq 0$, then $ A^{-1} = \frac{1}{\Delta_1}$. * If $ A \neq 0$, then $ A^{-1} = \frac{1}{\Delta_1}$. * If $ A = 0$ an upper or lower triangle matrix ¹ , then the determinant is the product of the diagonals. If one row or column consists of only zeros, then $ A = 0$. . If two rows or columns consists of only zeros, then $ A = 0$. . If we rows or columns consists of only zeros, then $ A = 0$. . If we rows or columns consists of only zeros, then $ A = 0$. . A las in proto columns. . $ A = 0$ • $ A \neq 0$ • If $ A = 0$ it is called singular, otherwise it is nonsingular. 9.06 Craner's Rule For the $n \times n$ matrix $ A $ with $ A \neq 0$, denote by A , the matrix obtained from A by replacing its it is column with the column vector b. Then the the component of the solution of the system is given by: we $x_{\perp} = \frac{ A_{\perp} }{ A }$ (c)(8) 10 Vector Spaces and Subspaces A vector space Y is a non-empty collection of elements that we call vec- tors, for which we can define the operation of vector addition and solars multiplications: 1. Addition: $x + y^2$ 2. Scalars: x^{0} where z is a constant. we than satisfy the following properties:	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} & b_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} & b_n \end{bmatrix} $ (14) 1. $\vec{x} + \vec{y} \in \mathcal{V}$ 2. $c\vec{x} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{x} + d\vec{y} \in \mathcal{V}$ which is called closure under linear combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{x} + \vec{y} \in \mathcal{V} - \text{Adimion}$ 2. $c\vec{x} \in \mathcal{V} = \text{Solar Multiplication}$ 3. $\vec{x} + \vec{0} = \vec{x} - \text{Zero Element}$ 4. $\vec{x} + (-\vec{x}) = (-\vec{x}) + \vec{x} = \vec{0} \leftarrow \text{Adimive Inverse}$ 5. $(\vec{x} + \vec{y}) = \vec{x} = \vec{x} + (\vec{y} + \vec{x}) \leftarrow \text{Adimive Property}$ 6. $\vec{x} + \vec{y} = \vec{y} + \vec{x} - \text{Commutativity}$ 7. $1 \cdot \vec{x} = \vec{x} + \text{Identity}$ 8. $c(\vec{x} + \vec{y}) = d + c\vec{y} \leftarrow \text{Distributive Property}$ 10. $c(d\vec{x}) = (a)\vec{x} \leftarrow \text{Associativity}$ 10.2 Vector Function Space A vector function space is just a unique vector space where the elements of	 1. Transform to RREF (??) using elementary row operations. 1. The linear matrix formed by this process has the same solutions as the final system, however it is much casier to solve. 50.2.2.2 Prominent Vector Punction Spaces R² → The space of all ordered pairs. R³ → The space of all ordered pairs. R³ → The space of all ordered rupples. R³ → The space of all ordered rupples. R⁴ → The space of all ordered rupples. R⁴ → The space of all polynomials with degree ≤ n. M_{ma} → The space of all continuous functions on the interval <i>I</i> (open, closed, finite, and infinite). C⁴(<i>I</i>) → Sime as above, except with n continuous derivatives. C⁴ → The space of all ordered n-tuples of complex numbers. C⁴(<i>I</i>) → Sime as above, except with n continuous derivatives. C⁴ → The space of all ordered n-tuples of complex numbers. Distribution of the space of all ordered n-tuples of complex numbers. If d, G ∈ W, than di + V̄ ∈ W. If d ∈ W and c ∈ ℝ, than c di ∈ W. If d ∈ W and c ∈ ℝ, than c di ∈ M. If d ∈ W and c ∈ ℝ, than c di ∈ M. If d ∈ W and the to a begace. All subspaces are vector spaces are using account of the stabove does not imply subspace. All subspaces are using account of the stabove concerve, where we check the closure with the above three. The tare only a couple subspaces R²: 	• $(A^{-1})^{-1} = A$ • A and B are invertible matrices of the same order if $(AB) = A^{-1}B^{-1}$ • If A is invertible than to is A^{T} and $(A^{-1})^{T} = (A^{T})^{-1}$ We can call the zero and the set V themselves trivial subspaces, calling the f subspace of lines passing through the origin the only non-trivial subspaces in \mathbb{R}^{2} . We can classify \mathbb{R}^{3} similarly: • Trivial: – Zero subspace – \mathbb{R}^{3} Non-Trivial – Lines that contain the origin. – Phaces that contain the origin. 10.3.1 Examples • The set of all solutions to $y'' - y't + y = 0$. • $\{P \in \mathbb{P}, P(2) = P(3)\}$ 11 Span, Basis and Dimension 11.1 Span The span of $\{y, y_{2}, \dots, y_{n}\}$ of vectors in a vector space \mathbb{V} , denoted by Span $\{\psi, \psi_{2}, \dots, \psi_{n}\}$ is the set of all linear combinations of the vectors. 11.1 Example For example, if $f = \begin{bmatrix} 2\\ 2 \end{bmatrix}$ and $\overline{\Psi} = \begin{bmatrix} 0\\ 2 \end{bmatrix}$	products subtracted. This process is demonstrated below. $\begin{aligned} A &= \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{22} \end{bmatrix} (16) \\ A &= \begin{bmatrix} a_1 & a_{22} \\ a_1 & a_{23} \end{bmatrix} (16) \\ \hline A &= \begin{bmatrix} a_1 & a_{23} \\ a_{11} & a_{21} \\ a_{12} & a_{11} \end{bmatrix} \\ A &= \begin{bmatrix} a_{22} & a_{11} & a_{12} \\ a_{12} & a_{11} \\ a_{12} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{22} & a_{11} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{11} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{11} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} & a_{23} \\ a_{23} & a_{23} & a_{23} \end{bmatrix} \\ A &= \begin{bmatrix} a_{23} & a_{23} & a_{23} & a_{23} \\ $
and rabbias are present, the number of interactions is ∞ the product of M $\left\{ \begin{array}{lll} \frac{M}{M} = a_{ij} R - c_{jk} RF \\ \frac{M}{M} = a_{ij} R - a_{ij} RF \\ \frac{M}{M} = AF \\$	* 8.2 Addition and Multiplication 1) Each new element in the matrix is a result of the dat product between the corresponding row and column matrices. 4 $ A ^2 = A$ $ A \neq 0$, then $ A^{-1} = \frac{1}{n}$; * If $A $ is an upper observ training matrix ⁴ , then the determinant is the product of the diagonals. I flow row or column consists of only zeros, then $ A = 0$. I flow row or column consists of only zeros, then $ A = 0$. I flow row or column consists of only zeros, then $ A = 0$. I flow row or column consists of only zeros, then $ A = 0$. I flow row or column consists of only zeros, then $ A = 0$. I flow row or column are equal, then $ A = 0$. I flow row or columns are equal, then $ A = 0$. I flow is invertible. 9 $ A = 0$ 10 $ A = 0$ is called singular, otherwise it is nonsingular. 9.64 Cramer's Rule For the n × n matrix A with $ A \neq 0$, denote by A, the matrix obtained from A by rejactions of the system is given by. π $x_{i} = \frac{ A }{ A }$ (18) A vector space A is a non-encoderised relation of the solution of the system. 1 Addition: $x + y$ 2. Scalars: cf where is a constant. * that antisfy the following propertien: $\frac{1}{r_{i}}$ that give during the solution the were there the lower are upper half is more effective.	a new matrix. $\begin{bmatrix} \mathbf{A} \mathbf{b} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} & b_1 \\ \vdots & \vdots & \ddots & \vdots \\ A_{41} & A_{42} & \cdots & A_{4n} & b_1 \\ \vdots & \vdots & \ddots & \vdots \\ A_{41} & A_{42} & \cdots & A_{4n} & b_n \end{bmatrix} $ (14) 1. $\vec{x} + \vec{y} \in \mathcal{V}$ 2. $c\vec{x} \in \mathcal{V}$ which can be condensed into a single equation: $c\vec{x} + d\vec{y} \in \mathcal{V}$ which is called closure under linar combinations. 10.1 Properties We have the properties from before, as well as new ones. 1. $\vec{x} + \vec{y} \in \mathcal{V} - \text{Addition}$ 2. $c\vec{x} \in \mathcal{V}$ Scalar Multiplication 3. $\vec{x} + \vec{0} = \vec{x} - \text{Zero Element}$ 4. $\vec{x} + (-\vec{x}) = (-\vec{x}) + \vec{x} = \vec{0} - \text{Additive Inverse}$ 5. $(\vec{x} + \vec{y}) + \vec{x} = \vec{x} + (\vec{y} + \vec{x}) \leftarrow \text{Additive Inverse}$ 5. $(\vec{x} + \vec{y}) = \vec{x} + \vec{x} - \text{Commitativity}$ 7. $1 \cdot \vec{x} = \vec{x} - \text{Identity}$ 8. $c(\vec{x} + \vec{y}) = d\vec{x} + d\vec{x} \leftarrow \text{Distributive Property}$ 9. $(c + d)\vec{x} = c\vec{x} + d\vec{x} \leftarrow \text{Distributive Property}$ 10. $c(d\vec{x}) = (ad)\vec{x} \leftarrow \text{Associatively}$ 10.2 Vector Function Space is an imple vector space where the elements of the space are functions. Note, the solutions to linear and homogeneous differential equations form	 3. Transform to RREF (??) using elementary row operations. 4. The linear matrix formed by this process has the same solutions as the initial system, however it is much casier to solve. 50.2.2. Prominent Vector Function Spaces R³ → The space of all ordered pairs. R³ → The space of all ordered n-tuples. R → The space of all continuous functions on the interval I (open, identify the space of all ordered n-tuples of ordered n-tuples. C[*](I) → The space of all continuous functions on the interval I (open, identify the space of all ordered n-tuples of complex numbers. C[*](I) → Same as above, except with n continuous dirictives. C[*] → The space of all ordered n-tuples of complex numbers. DAS Vector Subspaces The development addition and scalar multiplication: If if q ∈ W, uhan if + V ∈ W. If if q ∈ W and a h ∈ R, than all + hθ ∈ W. (20) Nate, vector space does not imply subspace. All subspaces are vector spaces. The ordered not imply subspace. The determine if it is a subspace, we check for closure with the above theorem. 	 (A⁻¹)⁻¹ = A A and B are invertible matrices of the same order if (AB) = A⁻¹B⁻¹ If A is invertible, then no is A^T and (A⁻¹)^T = (A^T)⁻¹ We can called by the avera of the set V themselves trivial subspace. Calling the function of the set V themselves trivial subspace. Trivial: Zero subspace R³ Non-Trivial Lines that contain the origin. Places that contain the origin. Places that contain the origin. IDe set of all even functions. The set of all even functions. (P ∈ R; P(2) = P(3)) 11 Span, Basis and Dimension 11.4 Span U The set of (9, 9,, 4) of vectors in a vector space V, denoted by Span(§1, \$v_1,, \$v_1\$) is the set of all incer combinations of the vectors. 11.1 Example 	products subtracted. This process is demonstrated below. $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_3 \end{bmatrix} \qquad (16)$ $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_3 & a_4 \end{bmatrix} \qquad (16)$ 11.1 2 Spanning Sets in R³ A vector b in R ⁵ set in R ³ as the set of the set o

•	A is invertible. A has n pivot columns. $ A \neq 0$		11.6.1	Standard Basis for \mathbb{R}^n	$\{\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n\}$ where
Then the of element	e set \vec{v} is linearly dependent	$\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\} \in \mathbb{R}^n$, dim $(\vec{v}) = m$ if $n > m$ where n is the number prove the opposite. It only goes one $\begin{pmatrix} 1\\ -3\\ 7 \end{pmatrix}$ Is dependent	are the	$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ column vectors of the identity m	
3. Columns the trivia	s of A are linearly independ al solutions of n .	lent if and only if $A\vec{\mathbf{x}}=\vec{0}$ has only	A vector The	Example r space can have different bases. standard basis for \mathbb{R}^n	is: $\{\vec{e}_1, \vec{e}_2\}$ for $\vec{e}_1 =$
One way to c	ear Independence of Fu heck a set of functions is to	consider them as a one dimensional	But a	nd $\vec{e}_2 \begin{bmatrix} 0\\1 \end{bmatrix}$ giving $\left\{ \begin{bmatrix} 1\\0 \end{bmatrix}, \begin{bmatrix} 0\\1 \\ 1 \end{bmatrix} \right\}$ nother basis for \mathbb{R}^2 is given by: $\left], \begin{bmatrix} 1\\2 \end{bmatrix} \right\}$]}
vector. $\vec{\mathbf{v}}_i(t)$	$= f_n(t)$ Another method is	the Wronskian:		Dimension of the Colur	nn Space of a Matrix
To	find the Wronskian of func	tions f_1, f_2, \dots, f_n on I : f_1, f_2, \dots, f_n		lly, the number of vectors in a b	
	$W[f_1, f_2,, f_n] =$	$\begin{bmatrix} f_1 & f_2 & \cdots & r_n \\ f_1' & f_2' & \cdots & r_n' \\ \vdots & \vdots & \ddots & \vdots \\ f_1^{n-1} & f_2^{n-1} & \cdots & r_n^{n-1} \end{bmatrix}$ (21)	• Th	Properties e pivot columns of a matrix A fo	
If $W \neq 0$ for the function s	all t on the interval I , we space is a linearly independence	here $f_1, f_2,, f_n$ are defined, then lent set of functions on I .	of p	e dimension of the column space, sivot columns in A. rank $A = din$ Invertible Matrix Character	
11.6 Bas	sis of a Vector Space	B	Let A b	e an $n \times n$ matrix. The following	
	$\vec{v}_2,, \vec{v}_n$ } is a basis for ve			s invertible. • column vector of A is linearly i	ndenendent
. (a. a	\dots, \vec{v}_n } is linearly independent	lant		ry column of A is a pivot column	
		icit.		e column vectors of A form a bas	is for $Col(A)$.
 Span{\$\vec{v}_1\$ Case One 	$\{\vec{v}_2, \dots, \vec{v}_n\} = V$			nk $A = n$	
$\Delta > 0$	Real Unequal Roots $r_1, r_2 = \frac{-b \pm \sqrt{D-4ac}}{2a}$	Overdamped Motion $y_h(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$	$c_n y_n(t)$) is a solution of L(y) = f _i (t), the solution of L(y) = c ₁ f ₁ (t) + e e to apply this, we need the no	then $y(t) = c_1y_1(t) = c_2y_2(t) + \cdots + c_nf_n(t)$ $c_2f_2(t) + \cdots + c_nf_n(t)$
Case Two $\Delta = 0$	Real Repeated Root $r = -\frac{b}{2a}$	Critically Damped Motion $y_h(t) = c_1 e^{it} + c_2 t e^{it}$		m 5 (Non-Homogeneous Princip	
Case Three $\Delta < 0$	e Complex Conjugate Roots $r_1, r_2 = \alpha \pm \beta i$ $\alpha = -\frac{b}{2\alpha}, \beta = \frac{\sqrt{4ac-b^2}}{2a}$	Underdamped Motion $y_h(t) = e^{\alpha t} \left(c_1 \cos \left(\beta t\right) + c_2 \sin \left(\beta t\right) \right)$	to ident particul are iden	ify the form of the particular s ar solution itself. Once the part tified, add them to determine the	making educated guesses in order olution, as well as eventually the icular and homogeneous solutions solution. The following table may
Table 1: Roc Equation For	ots for Second Order Differ	rential Equations in Characteristic	help ide $f($	ntify common formats and soluti t)	y _p (t)
	ear Independence		1 k	.(<i>t</i>)	$A_0 = A_0(t)$
The Solution	Space Theorem (??) provid	des us with the number of solutions	3 C 4 C	e^{kt} $\cos(\omega t) + D \sin(\omega t)$	$A_0 e^{kt}$
	an nth order homogeneou		5 P, 6 P.	$a(t)e^{kt}$ $a(t)\cos(\omega t) + Q_n(t)\sin(\omega t)$ $e^{kt}\cos(\omega t) + De^{kt}\sin(\omega t)$	$\begin{array}{l} A_0 \cos(\omega t) + B_0 \sin(\omega t) \\ A_n(t) e^{kt} \\ A_n(t) \cos(\omega t) + B_n(t) \sin(\omega t) \\ A_0 e^{kt} \cos(\omega t) + B_0 e^{kt} \sin(\omega t) \end{array}$
	with <i>m</i> solutions for the <i>n</i> t be independent.	th order case, if $m > n$ the solutions	7 C 8 P	$e^{kt} \cos(\omega t) + De^{kt} \sin(\omega t)$ $_{a}(t)e^{kt} \cos(\omega t) + Q_{a}(t)e^{kt} \sin(\omega t)$	$A_0 e^{kt} \cos(\omega t) + B_0 e^{kt} \sin(\omega t)$ $A_n(t) e^{kt} \cos(\omega t) + B_n(t) e^{kt} \sin(\omega t)$
. If m = m					
	a, we must test using the co	-		Table 2: Guesses for Pa	rticular Solutions
 If m < n 	a, the set does not span the	-	 P_n(rticular Solutions
 If m < n 12.4.1 Wre The Wronskii 	a, the set does not span the onskian an also tells us about the li	space.	 A₀, 	Table 2: Guesses for Pa $t), Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 \equiv \mathbb{R}$	rticular Solutions
 If m < n 12.4.1 Wro The Wronskii tions. This W Suppose {g differential eq 	a, the set does not span the onskian an also tells us about the li y_{1} , y_{2} ,, y_{n} } is a set of solv	space. inear independence of a set of func Wronskian previously defined (??) itions of an <i>n</i> th order homogeneous	 A₀, k, ω In 	$(t), Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 \equiv \mathbb{R}$ $(t), C, D \in \mathbb{R}$	rticular Solutions t t be in y_p even if only one term is
 If m < n 12.4.1 Wro The Wronskii tions. This W Suppose {g differential eq L(y) = a_n(1. If W[y₁, 	a, the set does not span the onskian an also tells us about the li <i>i</i> vronskian is identical to the y_1, y_2, \dots, y_n is a set of solu- quation. (1) $y^n + a_{n-1}(1)y^{n-1} + \dots + i$ $y_2, \dots, y_n \neq 0$ at any poin	space. inear independence of a set of func Wronskian previously defined (??) itions of an <i>n</i> th order homogeneous	 A₀, k, ω In pre If any 	$t), Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 \equiv \mathbb{R}$ $i, C, D \in \mathbb{R}$ $\frac{1}{4}$ and $\overline{\mathbb{G}} - \underbrace{\mathbb{S}}$ both terms muss sent in $f(t)$.	
 If m < n 12.4.1 Wred The Wronskii tions. This W Suppose {y differential ec L(y) = a_n(If W[y₁, independent If W[y₁, independent 	, the set does not span the onskian an also tells us about the li Vronskian is identical to the $\eta_1, \eta_2, \dots, \eta_k$] is a set of soli $\eta_1y^n + a_{n-1}(t)y^{n-1} + \dots + t,$ $\eta_2, \dots, \eta_k \neq 0$ at any poir lent. $\eta_2, \dots, \eta_k = 0$ at every poir	space. inear independence of a set of func Wronskian previously defined (??) ations of an nth order homogeneous $a_1(t)y' + a_0(t)y = 0$	 A₀, k, ω In pre If any eliminat 	$t), Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 \equiv \mathbb{R}$ $c, C, D \in \mathbb{R}$ $\frac{1}{2}$ and $[6] - [8]$ both terms mus sent in $f(t)$. term or terms of y_p is found in the duplication.	t be in y_p even if only one term is $y_{0,*}$ multiply the term by t or t^2 to
 If m < n 12.4.1 Wro The Wronskii tions. This W Suppose { differential eq L(y) = a_n(If W[y₁, independential operations) If W[y₁, dependential operations) 	the set does not span the onskinn an also tells us about the li- lyronskinn is identical to the $(y_{1}, y_{2}, \dots, y_{n})$ is a set of solu $(y_{1}^{n} + a_{n-1}(t)y^{n-1} + \dots + y_{2}, \dots, y_{n}) \neq 0$ at any point $y_{2}, \dots, y_{n} \neq 0$ at any point $y_{2}, \dots, y_{n} = 0$ at every point.	space. inear independence of a set of func Wronskian previously defined (77) itions of an with order homogeneous $a_i(t)y' + a_0(t)y = 0$ it on (a, b) , then the set is linearly int on (a, b) , then the set is linearly	 A₀, k, a In [pre If any eliminat 12.6 We've a 	t), $Q_n(t)$, $A_n(t)$, $B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 \equiv \mathbb{R}$; $C, D \in \mathbb{R}$ \exists and $[\overline{0} - [\overline{S}]$ both terms mus- sent $\overline{m} f(t)$. term or terms of y_t is found in the duplication. Variation of Parameter ineady used variation of parameter	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to S s t ers to find the solutions of $u' + t$
 If m < n 12.4.1 Wro The Wronskii, tions. This W Suppose f_k differential co L(y) = a_n(If W[y₁, independential If W[y₁, independential If W[y₁, independential If W[y₁, independential 	the set does not span the onskian an also tells us about the li (rouskian is identical to the $p_1, p_2,, p_k$) is a set of soli puttion. $t)y^n + a_{n-1}(t)y^{n-1} + + ,$ $p_2,, p_k \neq 0$ at any point lent. $p_2,, p_k = 0$ at every point. determined Coefficial $L(y) = a_n(t)y^n + a_{n-1}(t)y^n$	space. inear independence of a set of func Wronskian previously defined (77) itions of an with order homogeneous $a_i(t)y' + a_0(t)y = 0$ it on (a, b) , then the set is linearly int on (a, b) , then the set is linearly	 A₀, k, ω In [pre If any eliminat 12.6 We've a p(t) = in the fic w/r + p'' + p''	(i) $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$; $C, D \in \mathbb{R}$] and $(b) - (b)$ both terms mus- ent in $f(t)$. term or terms of a_0 is found in the duplication. Variation of Parameter heady used variation of param- term: f(t). This same strategy can be error.	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s terms to find the solutions of $y' +$ applied to second order equations
 If m < n It m < n 12.4.1 Wre The Wronskii tions. This W Suppose {b differential ec L(y) = a_n(If W[y₁, independential If W[y₁, dependential If W[y₁, dependential If W[y₁, dependential Unot Let's assume t ∈ some interview 	the set does not span the onskian an also tells us about the li (rouskian is identical to the $p_1, p_2,, p_k$) is a set of soli puttion. $t)y^n + a_{n-1}(t)y^{n-1} + + ,$ $p_2,, p_k \neq 0$ at any point lent. $p_2,, p_k = 0$ at every point. determined Coefficial $L(y) = a_n(t)y^n + a_{n-1}(t)y^n$	space. insear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(t)y' + a_0(t)y = 0$ int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents $e^{-1} + \dots + a_1(t)y' + a_0(t)y = 0$ where	 A₀, k, ω In pre- If any eliminat 12.6 We've a p(l) y = in the fe y" + p To ap 	(t), $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 \equiv \mathbb{R}$ $; C, D \in \mathbb{R}$ \exists and $(0) - [S]$ both terms mus- sent in $f(t)$. Variation of Parameter term variation of parameter trudy used variation of parameter trudy used variation of parameter trudy used variation of parameter trudy used variation of parameter trudy variation of parameter	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s terms to find the solutions of $y' +$ applied to second order equations ps.
• If $m < n$ 12.4.1 Wrot The Wronski tions. This W Suppose $\{y \ differential e \ L(y) = a_n$ ($1 \ HW[y_n, independential of the line of the $, the set does not span the onskinn an also tells us about the li ('onskian is identical to the y_1, y_2, \dots, y_k) is a set of solu $(y^n + a_{n-1}(t)y^{n-1} + \dots + t_k)$ (y_1, y_1, \dots, y_k) = 0 at any point leat. y_2, \dots, y_n = 0 at every point. determined Coefficie $L(y) = a_n(t)y^n + a_{n-1}(t)y^n$ real L	space. inear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(t)y' + a_0(t)y = 0$ it on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents $e^{-1} + \cdots + a_1(t)y' + a_0(t)y = 0$ where netric)	 A₀, k, α In [pre If any eliminat 12.6 We've a p(t)y = in the fc y" + p To ap 1314.3.1 	$(t), Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_g \equiv \mathbb{R}$ $\zeta, C, D \in \mathbb{R}$ $\exists \text{ and } \bigcup_{i=1}^{n-1} Siboth terms muss set in f(t). So both terms of g_g is found inthe dashpitchicusVariation of Parameterf(t)$. This same strategy can be $d(t) \neq n(t) = f(t)$ gives the strategy of the set	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s terms to find the solutions of $y' +$ applied to second order equations ps.
• If $m < n$ 12.4.1 Wr The Wronskii tions. This W Suppose (), differential ec $L(y) = a_n(x)$ depende 12.5 Un Let's assume $t \in$ some inter • Equilibri • Speed is	, the set does not span the onskinn an also tells us about the li by (p_1, \dots, p_k) is a set of solid (p_1, \dots, p_k) is a set of solid $(p_1, \dots, p_k) = 0$ at any point letter. $(p_1, \dots, p_k) = 0$ at any point $(p_1, \dots, p_k) = 0$ at any point $(p_1, \dots, p_k) = 0$ at every point at $L(y) = a_k(t)y^{\mu} + a_{n-k}(t)y^{\mu}$ ummarives at origin (Symmi	space. inear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(1y' + a_0(t)y = 0$ at on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents ents netric) of the eigenvalues.	• A_0 , • k, ω • $In [$ pre If any eliminat 12.6 We've a p(t)y = in the fit y'' + p To ap 1314.3.1 $\begin{bmatrix} \vec{x}_r \\ \vec{x}_i \end{bmatrix} =$	(i), $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 \equiv \mathbb{R}$ $i, C, D \in \mathbb{R}$ \exists and $i \in [0, -]$ shoth terms mass set in $f(t)$. term or terms of u_j is found in the duplication. Variation of Parameter heady used variation of param f(t). This same strategy can be $i(f_1) - i(h) = i(h) = i(h)$.	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s terms to find the solutions of $y' +$ applied to second order equations ps. nvalues
• If $m < n$ 12.4.1 Wr The Wronskii tions. This W Suppose () differential ec $L(y) = a_n()$ 1. If $W[y_{P_1}, \dots, y_{P_n}]$ depender 12.5 Unc 12.5 Unc 12.5 Unc 12.5 Unc 12.5 Unc 13.5 Speed is 14.2 Line To solve a say	, the set does not span the onskin an also tells us about the li $y_1, y_2, \dots, y_n > $ is a set of soir $y_1, y_1, \dots, y_n > $ is a set of soir $y_1, y_1, \dots, y_n > $ of at any point lett. $U(y) = a_n(y)y^n + a_{n-1}(y)y^{n-1}$ thus arrive a origin (Symu determined by magnitude	space. inear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(1y' + a_0(t)y = 0$ at on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents ents netric) of the eigenvalues.	• A_0 , • k, ω • $\ln \left[\text{prec} \right]$ If any eliminat 12.6 We've a $p(t)y = \frac{1}{2}$ in the fx y'' + p To ap 1314.3.1 $\left[\vec{x}_r \\ \vec{x}_i \right] =$ • The	(i) $Q_n(t) A_n(t) B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$ $C, D, E \in \mathbb{R}$ $\exists \text{ and } \widehat{\mathbb{B}}_{-1} = \widehat{\mathbb{S}}$ both terms must even in $f(0)$. term or terms of y_0 is found in the duplication. Variation of Parameter heady variation of parameters the duplication of parameters $f(0)^{-1} \oplus (3 \operatorname{sm} \mathbb{R}^n) = f(0)$ by this method, follow these stat Interpreting Non-Real Elige $e^{-dt} \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ (\beta) & -\cos(\beta) & -\sin(\beta) \end{bmatrix} \begin{bmatrix} \widetilde{\mathbf{q}} \\ \widetilde{\mathbf{q}} \end{bmatrix}$ if first variable defines the exposite $f(\alpha > 0 \to 0 \operatorname{Coverlu} without be$	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s sets to find the solutions of $y' +$ applied to second order equations ps. nvalues sion.
• If $m < n$ 12.4.1 Wr The Wronski tions. This W Suppose $\{n$ differential $e \in L(p) = a_n(1 - p) = a$, the set does not span the onskinn an also tells us about the li h_1, h_2, \dots, h_n is a set of soli $y_1 = y_1, \dots, y_n$ is a set of soli $y_1 = y_1, \dots, y_n$ is 0 at solve $y_1 = y_1, \dots, y_n$ determined Coefficit determined Coefficit $L(y) = a_1(x)y^n + a_{n-1}(y)y^n$ immarives at origin (Symu determined by magnitude ear Systems with R	space. inear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(1y' + a_0(t)y = 0$ at on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents ents netric) of the eigenvalues.	• A_0 . • k, α • $\ln \left[\text{pre} \right]$ If any eliminat 12.6 We've a $p(t)y = \frac{1}{2}$ in the fx y'' + 1 To ap 1314.3.1 $\left[\vec{x}_r \\ \vec{x}_r \right] =$ • The	(i), $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathcal{P}_0 \equiv \mathbb{R}$ $i, C, D \in \mathbb{R}$] and $(\overline{0}) - [\overline{S}]$ both terms mus- serin $f(t)$. term or terms of y_0 is found in i the duplication. Variation of Parameter: Invalues a strategy can be same f(t). This same strategy can be same f(t) = (1) + (1)	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s sets to find the solutions of $y' +$ applied to second order equations ps. nvalues sion.
• If $m < n$ 12.4.1 Wr The Wronski tions. This W Suppose $\{y \in L(y) = a_n(x) + (y + 1) + $, the set does not span the onskin an also tells us about the li (p_1, p_2, \dots, p_n) is a set of soir (p_1, p_2, \dots, p_n) is a set of soir $(p_1, p_2, \dots, p_n) = 0$ at any poin leaf. $(p_1, \dots, p_n) = 0$ at any poin $(p_1, \dots, p_n) = 0$ at every point determined Coefficit $L(y) = a_n(y)y^n + a_{n-1}(y)y^{n-1}$ hum arrives at origin (Symu determined by magnitude ease Systems with R stem in the form	space. inear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(1y' + a_0(t)y = 0$ at on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents ents netric) of the eigenvalues.	• A_0 , • k, α • In pre- If any eliminat 12.6 We've a $p(t)y =$ in the fc y'' + p To ap 1314.3.1 $\begin{bmatrix} \vec{x}_r \\ \vec{x}_i \end{bmatrix} =$	(i), $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$ \vdots , $C, D \in \mathbb{R}$ \exists and $(\underline{b}) - [\mathbf{S}]$ both terms mus- servin $I(t)$. term or terms of u_j is found in the duplication. Variation of Parameter: heady used variation of parameter: heady used variation of Parameter: I(t) = The same state (t) = The same sta	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s sets to find the solutions of $y' +$ applied to second order equations ps. nvalues sion.
• If $m < n$ 12.4.1 Wro The Wronski tions. This W Suppose $(g$ differential e_{1} $L(g) = a_{0}$ 12.5 Unc 12.5 Unc 12.5 Unc 12.6 Unc 13.6 Solution 13.6 Solution 3. Solution	the set does not span the onskinn an also tells us about the li- hyperbolic constants is derived by the hyperbolic constant of the set of solid partial and the set of solid partial and the set of the set of the left of the set of the set of the set of the determined Coefficie L(y) = a_1(x)y^2 + a_{-1}(y)y^2 determined by magnitude constant of the set of the set of the determined by magnitude constant of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the envalues of A. constant eigenvectors.	space. inear independence of a set of func Wronskian previously defined (??) iters of an nth order homogeneous $a_i(1y' + a_0(t)y = 0$ at on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents ents netric) of the eigenvalues. eal Eigenvalues matrix at least) our solution is in	• A_{0} , • k, α • In [pre- If any eliminat 12.6 We've as $p(t)y = f(t)$ in the for W or α is p(t)y = f(t) in the form $f(t) = f(t)$ in the form $f(t) = f(t)$ in the form $f(t) = f(t)$ in the form $f(t) = f(t)$ • The form $f($	(i), $Q_{i}(t), A_{i}(t), B_{i}(t) \in \mathbb{P}$ $B_{ij} \in \mathbb{P}_{g} \equiv \mathbb{R}$ $i, C, D \in \mathbb{R}$ $\exists and [1] - [5]$ both terms mus- sent in $f(t)$. Terms of g_{ij} is found in the daphization. Variation of Parameter heady used variation of param- f(t). This same strategy can be m_{ij} . This same strategy can be m_{ij} . This same strategy can be m_{ij} . $g_{ij}(t) = f(t)$ by this method, follow these set Laterpreting Non-Real Eigen $= e^{i t t} \begin{bmatrix} \cos(\alpha) - \sin(\alpha) \\ \sin(\beta) + \cos(\beta) \end{bmatrix} \begin{bmatrix} \widetilde{\mathbf{q}} \\ \widetilde{\mathbf{q}} \end{bmatrix}$ first variable defines the expanse. If $\alpha < 0 \rightarrow$ Decay to 0 . $:$ If $\alpha = 0 \rightarrow$ Period solutions. second defines rotation. : Constructed i < 0 $j < 0$	t be in y_p even if only one term is y_p , multiply the term by t or t^2 to s sets to find the solutions of $y' +$ applied to second order equations ps. nvalues sion.
• If $m < n$ 12.4.1 Wro The Wronski tions. This W Suppose $(g$ differential e_{1} $L(g) = a_{0}$ 12.5 Unc 12.5 Unc 12.5 Unc 12.6 Unc 13.6 Solution 13.6 Solution 3. Solution	, the set does no span the onskinn an also tells us about the li $(r_{1}, r_{2},, r_{k}) = 0$ as set of solid $(r_{1}, r_{k}),, r_{k}) = 0$ at any tori- lead. $(r_{1}, r_{k}),, r_{k}) = 0$ at any tori- lead. $(r_{1}, r_{k}),, r_{k}) = 0$ at any tori- term of the solid result of the solid result of the solid $(r_{1}, r_{k}) = a_{k}(r_{1})r^{k} + a_{k-1}(r_{1})r^{k}$ $(r_{k}, r_{k}) = a_{k}(r_{k})r^{k} + a_{k-1}(r_{k})r^{k}$ $(r_{k}, r_{k}) = a_{k}(r_{k})r^{k} + a_{k-1}(r_{k})r^{k}$	space. inear independence of a set of func Wronskian previously defined (??) iters of an nth order homogeneous $a_i(1y' + a_0(t)y = 0$ at on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents ents netric) of the eigenvalues. eal Eigenvalues matrix at least) our solution is in	• A_{0} , • k, α • In [pre- If any pre- imitation of the second of the second pre-	(i), $Q_{i}(t), A_{i}(t), B_{i}(t) \in \mathbb{P}$ $B_{ij} \in \mathcal{P}_{ij} \equiv \mathbb{R}$ $C, D \in \mathbb{R}$ $\exists a \in \mathbb{C}_{ij} = \S$ both terms mus- serie in $f(t)$. So both terms mus- serie in $f(t)$. The same strategy can be the daplication: Variation of Parameter M(t). This same strategy can be $m_{int}^{(1)}$. This same strategy can be $m_{int}^{(2)}$. The same strategy can	t be in y_p even if only one term is g_p , multiply the term by t or t^2 to \mathbf{s} sters to find the solutions of $y' + a$ applied to second order equations ps. nvvalues ion. und.
• If $m < n$ 12.4.1 Wro The Wronski tions. This W Suppose b_i differential e_i $L(y) = a_i$ 13.5 Unc 12.5 Unc 12.5 Unc 12.5 Unc 12.6 Unc 12.6 Unc 12.6 Unc 12.6 Unc 12.6 Unc 12.6 Unc 12.6 Unc 12.6 Unc 12.6 Unc 13.8 Unc 13.9 Unc 14.9 Unc 15.9 Colored 3. Solution 16.6 form In there are	the set does not span the onskin an also tells us about the li (p_1, p_1, \dots, p_k) is a set of soir $(p_1, p_2, \dots, p_k) = 0$ at any point lent. $(p_1, p_2, \dots, p_k) = 0$ at any point lent. $(p_1, p_2, \dots, p_k) = 0$ at any point $(p_1, p_2, \dots, p_k) = 0$ at every point determined Coeffici $L(p) = a_k(1)p^k + a_{k-1}(1)p^{k-1}$ mum arrives at origin (Symm determined by magnitude ease Systems with R stem in the form envalues of A . concitated eigenvectors. is in the form (or $a \ge x \ge x$ $s(1) = a_k(1)^k = a_k^{k-1}q^{k-1}q^{k-1}$ $s(2) = a_k^{k-1}q^{k-1}$	space. inear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(1y' + a_0(1)y = 0$ it on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents $e^{-1} + \cdots + a_1(1y' + a_0(1)y = 0$ where netric) of the eigenvalues. eal Eigenvalues entatrix at least) our solution is in	• A_0 , • k, ω • k, ω If any eliminat 12.6 We've a y'' y = y'' in the $f'' = y''' y = y''''''''''''''''''''''''$	(i), $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$; $C, D \in \mathbb{R}$] and $(\underline{b}) - [\underline{b})$ both terms mus- series in $f(D)$. term or terms of a_0 is found in the charged variation of param- f(D). This same strategy can be the charged variation of param- f(D). This same strategy can be must be charged variation of param- f(D). This same strategy can be even: (D, This same strategy can be even: $(D, This same strategy can be even: (D, This same strategy can be even: (D, This same strategy can be even: (C, D, This same strategy can be even: (C, D, This same strategy can be even: (C, D, This same strategy can be even: (T, This same strategy can be (T, T, T, T, This same strategy can be (T, T, T, T, T, This same strategy can be (T, T, T$	t be in y _p even if only one term is y _p , multiply the term by t or t ² to s terms to find the solutions of y' + applied to second order equations pp. nvalues ion. und. nssification uillbrium solution. An equilibrium
• If $m < n$ 12.4.1 Wro The Wronski tions. This W Suppose $\{g$ differential eq $L(g) = a_n($ • $I + W [h]_n$, independential 12.5 Unc 12.5 Unc • $I + W [h]_n$, • $I + W [h]_n$, • $I = 0$ which is a • $I = 0$ which is a summary • $I = 0$ and in the form I find energy • $I = 0$ find as • $I = 0$ find the form I find the form • $I = 0$ find the form $I = 0$ for $I = 0$	the set does no span the onskin an also tells us about the li $y_1, y_2, \dots, y_n > $ is a set of soli $y_2, \dots, y_n > $ of a set of soli $y_1, \dots, y_n > $ of a set of soli $y_1, \dots, y_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of a set of soli $z_1, \dots, z_n > $ of $z_1, \dots, z_n > $ in an afficient of for a 2×2 $z_1 = z_1 - z_2 $	space. inear independence of a set of func Wronskian previously defined (??) itions of an nth order homogeneous $a_i(1y' + a_0(1)y = 0$ it on (a, b) , then the set is linearly int on (a, b) , then the set is linearly int on (a, b) , then the set is linearly ents $e^{-1} + \cdots + a_1(1y' + a_0(1)y = 0$ where netric) of the eigenvalues. eal Eigenvalues entatrix at least) our solution is in	• A_{0} , k, ω • k, ω • la fa pre- eliminat 12.6 We've a <i>s</i> <i>g'</i> (<i>ty</i>) = <i>i</i> i the <i>t</i> <i>g''</i> + <i>t</i> <i>g</i>	(i), $Q_{i}(t), A_{i}(t), B_{i}(t) \in \mathbb{P}$ $B_{ij} \in \mathbb{P}_{g} \equiv \mathbb{R}$ $i, C, D \in \mathbb{R}$ and $[\frac{1}{2}] - [\frac{1}{2}]$ both terms mus- sent in $f(t)$. The same strategy can be the daphication. Variation of Parameter heady used variation of param- (f_{ij}) . This same strategy can be m_{ij} . This same strategy can be m_{ij} . $(f_{ij}) = f(t)$ $(f_{ij}) = f(t)(f_{ij}) = f(t)(f_{ij})$ where $f(t) = f(t)(f_{ij})$ in first variable defines the expan- t for a circle $f(t) = co(f_{ij})$. $\left[\left[\frac{\tilde{q}}{2}\right]\right]$ if first variable defines the expan- $I \ \alpha < 0 \rightarrow Occup to 0$. $I \ \alpha < 0 \rightarrow Occup to 0$. $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$, $I \ \alpha < 0 \rightarrow Occup to 0$,	t be in y ₀ even if only one term is y ₀ , multiply the term by <i>t</i> or <i>t</i> ² to 8 even to find the solutions of y' + applied to second order equations ps. avalues ion. and. assification multiplytum solution. An equilibrium at.
• If $m < n$ 12.4.1 Wro The Wronski tions. This W Wronski tions. This W Wronski tions. This W (n) = n (n	the set does not span the onskin an also tells us about the li (p_1, p_1, \dots, p_k) is a set of soir $(p_1, p_2, \dots, p_k) = 0$ at any point lent. $(p_1, p_2, \dots, p_k) = 0$ at any point lent. $(p_1, p_2, \dots, p_k) = 0$ at any point $(p_1, p_2, \dots, p_k) = 0$ at every point determined Coeffici $L(p) = a_k(1)p^k + a_{k-1}(1)p^{k-1}$ mum arrives at origin (Symm determined by magnitude ease Systems with R stem in the form envalues of A . concitated eigenvectors. is in the form (or $a \ge x \ge x$ $s(1) = a_k(1)^k = a_k^{k-1}q^{k-1}q^{k-1}$ $s(2) = a_k^{k-1}q^{k-1}$	space. inear independence of a set of func Worskian previously defined (72) itors of an with order homogeneous $a(t)(y) = a_0(t) = 0$ at on (a, b) , then the set is linearly ents $a^{-1} + \cdots + a_4(t)y' + a_0(t)y = 0$ where netric) of the eigenvalues. eat Eigenvalues matrix at least) our solution is in (repeated eigenvalues), follow the	 A₀, a, b, a, b,	(i), $Q_i(t)$, $A_i(t)$, $B_i(t) \in \mathbb{P}$ $B_i \in \mathcal{P}_g \equiv \mathbb{R}$ $C, D \in \mathbb{R}$ $a_i \in \mathcal{P}_g = \mathbb{R}$ i , $C, D \in \mathbb{R}$ $a_i = 0$, $a_i = 0$, b_i both terms mus- serie in $f(t)$. This same strategy can be the daplication: Variation of Parameter $d(0) \neq u(0) = f(t)$ by this method, follow these set $d(0) \neq u(0) = f(t)$ $a_i = t_i = 0$, $a_i = 0$, $a_i = 0$, $a_i = 0$, $a_i = t_i = 0$, $a_i = $	t be in y_0 even if only one term is g_0 , multiply the term by t or t^2 to \mathbf{s} even to find the solutions of $y' + a_0$ presections presections and and and and and and and and
• If $m < n$ 12.4.1 Wro The Wronski tions. This Wronski differential to differential to differential to differential to dependential 2.5.1 Uncl Let's assume $t \in$ some inter $t \in$ some inter $t \in$ some inter $t \in$ some inter $t \in$ some inter 3.5.2 Uncl Let's assume $t \in$ some inter 5.5.2 Uncl Let's assume $t \in$ some inter 5.5.2 Uncl Let's assume $t \in$ some inter 5.5.2 Uncl Let's assume 5.5.2 Uncl Let's assume 5.5.2 Uncl 1.5.2 Uncl 1.5.2 Uncl 1.5.2 Uncl 1.5.2 Uncl 1.5.2 Uncl 1.5.3 U	, the set does not span the omkin man also tells us about the li type, the set of the set of the set of the set of the term of the set of the set of the set of the set of the groups of the set of the set of the set of the set of the groups of the set of	space. inear independence of a set of func Worskian previously defined (72) itors of an with order homogeneous $a(t)(y) = a_0(t) = 0$ at on (a, b) , then the set is linearly ents $a^{-1} + \cdots + a_4(t)y' + a_0(t)y = 0$ where netric) of the eigenvalues. eat Eigenvalues matrix at least) our solution is in (repeated eigenvalues), follow the	 A₀, a, b, a, b,	(i), $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$ $\zeta, C, D \in \mathbb{R}$ $\exists \text{ and } [\overline{B}] - [\overline{S}]$ both terms muss $i \in C, D \in \mathbb{R}$ $\exists \text{ and } [\overline{B}] - [\overline{S}]$ both terms muss term or terms of y_0 is found in the duplication. Variation of Parameter heady variation of parameters [1, C]. This same strategy can be rm: $(1, d) \neq q(t)y = f(t)$ by this method, follow these ste $[1, G = 0 \rightarrow Goog(h) = [1, G]$ $[1, G] = (cos(\beta) - sin(\beta) + [1, G])$ $[1, G] = (cos(\beta) - sin(\beta) + [1, G])$ $[1, G] = (cos(\beta) - sin(\beta) + [1, G])$ $[1, G] = 0 \rightarrow Goog(h)$ the equation $I: f \alpha = 0 \rightarrow Peroto solutions. c second defines rotation.c Constructed columbra \Xi \in G called an q othird defines thi and shape.Stability and Linear CIand solutions \Xi = 6 is called an q ob-obtions remain close and tred ticolumbra for \beta > 0$	t be in y ₀ even if only one term is y ₀ , multiply the term by <i>t</i> or <i>t</i> ² to 8 even to find the solutions of y' + applied to second order equations ps. avalues ion. and. assification multiplytum solution. An equilibrium at.
• If $m < n$ 12.4.1 Wro The Wronski tions. This Wronski differential ce differential ce $U(p) = \alpha(q)$. • $U(p) = \alpha(q)$. • $U(p)$, the set does not span the omkin an also tells on about the li formshain is identical to the matrix of the set of the set of the set of the matrix of the set of the	space. inear independence of a set of func Wooskian previously defined (17) itiums of an with order homogeneous $a_1(y) = a_0(1y) = 0$ at on (a, b) , then the set is linearly ents $a_1 + \dots + a_1(t)y' + a_0(t)y = 0$ where netric) of the eigenvalues. ena Eigenvalues entrix at least) our solution is in (repeated eigenvalues), follow the a_1 , $a_2 = a \pm i\beta$, the correct provides $\lambda_1, \lambda_2 = a \pm i\beta$, the correct	• A_{0} , A_{v} ,	(i), $Q_n(t), A_n(t), B_n(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$ $\zeta, C, D \in \mathbb{R}$ $\exists \text{ and } [\overline{B}] - [\overline{S}]$ both terms muss $i \in C, D \in \mathbb{R}$ $\exists \text{ and } [\overline{B}] - [\overline{S}]$ both terms muss term or terms of y_0 is found in the duplication. Variation of Parameter heady variation of parameters [1, C]. This same strategy can be rm: $(1, d) \neq q(t)y = f(t)$ by this method, follow these ste $[1, G = 0 \rightarrow Goog(h) = [1, G]$ $[1, G] = (cos(\beta) - sin(\beta) + [1, G])$ $[1, G] = (cos(\beta) - sin(\beta) + [1, G])$ $[1, G] = (cos(\beta) - sin(\beta) + [1, G])$ $[1, G] = 0 \rightarrow Goog(h)$ the equation $I: f \alpha = 0 \rightarrow Peroto solutions. c second defines rotation.c Constructed columbra \Xi \in G called an q othird defines thi and shape.Stability and Linear CIand solutions \Xi = 6 is called an q ob-obtions remain close and tred ticolumbra for \beta > 0$	t be in y_0 even if only one term is g_0 , multiply the term by t or t^2 to \mathbf{s} even to find the solutions of $y' + a_0$ presections presections and and and and and and and and
• If $m < n$ 12.4.1 Wro The Wronski tions. This Wronski tions. This Wronski tions. This Wronski tions. This Wronski Higher States of the time $\ell = 0$ and the time 13.5 Unc 13.5 Unc 14.5 Unc 15.6 Not 16.7 Not 17.6 Not	, the set does not span the onskinn an also tells us about the li- frequency of the set of the set of the set of the $(p_1, p_2, \dots, p_n) \neq 0$ at any topic least $(p_1, p_2, \dots, p_n) \neq 0$ at any topic least $(p_1, p_2, \dots, p_n) \neq 0$ at any topic determined Coefficient $(p_1, p_2, \dots, p_n) \neq 0$ at any topic determined Coefficient determined Set of the set o	space. inear independence of a set of funce Wronskian previously defined (1?) ittors of an anth order homogeneous $a_i(t)y' + a_i(t)y = 0$ it on (a, b) , then the set is linearly ents $a_1 + \dots + a_1(t)y' + a_i(t)y = 0$ where netric) of the eigenvalues. cal Eigenvalues. entrix at least) our solution is in (repeated eigenvalues), follow the $a_1(t)$, envalues $\lambda_1, \lambda_2 = \alpha \pm i\beta$, the corre- conjugate pairs in the form:	 A₀, A₁ A₁ A₁ A₁ A₁ A₁ A₂ A₁ A₂ A₁ A₁ A₂ A₁ A₂ A₁ A₂ A₁ A₂ A₁ A₂ A₁ A₁ A₁ A₁ A₁ A₁ A₁ A₁ A₁ 	(i), $Q_i(t), A_i(t), B_i(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$ $\zeta_i \subset D \in \mathbb{R}$ $\exists_i \in \mathbb{Q}_0 = \mathbb{R}$ both terms mus- term or terms of y_i is found in the duplication. Variation of Parameter heady variation of param- tic duplication of param- terms of y_i is found in the theorem of the duplication. Variation of Parameter Interpreting Non-Real Eigen $e^{-d} = \begin{bmatrix} \cos(\beta_i) & -\sin(\beta_i) \\ (\beta_i) + \cos(\beta_i) \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix}$ instructional bediefine the equation of the duplication of the duplicat	t be in y_0 even if only one term is y_0 , multiply the term by t or t^2 to S sets to find the solutions of $y' +$ applied to second order equations ps. nvalues dom. und. assification uilibrium solution. An equilibrium nt. o t as $t \to \infty$ we call this neutrally or repelled, we call this neutrally unster Plane
• If $m < n$ 12.4.1 Wro The Wronski tions. This W Suppose $(s$ differential e_{i} $L(s) = a_{i}$ • $L(s) = a$, the set does not span the onskinn an also tells us about the li- frequencies of the set of the set of the set of the $(y_1, y_2, \dots, y_n) = 0$ at any tori- lease. b (y_1) = a_{11}(y_1)^{n-1} + a_{1-1}(y_1)^{n-1} b (y_1) = a_{11}(y_1)^{n-1} + a_{1-1}(y_1)^{n-1} c (y_1) = a_{11}(y_1)^{n-1} + a_{1-1}(y_1)^{n-1} + a_{1-1}(y_1)^{n-1} c (y_1) = a_{11}(y_1)^{n-1} + a_{1-1}(y_1)^{n-1} +	space. inear independence of a set of func Wooskian previously defined (17) itiums of an with order homogeneous $a_1(y) = a_0(1y) = 0$ at on (a, b) , then the set is linearly ents $a_1 + \dots + a_1(t)y' + a_0(t)y = 0$ where netric) of the eigenvalues. ena Eigenvalues entrix at least) our solution is in (repeated eigenvalues), follow the a_1 , $a_2 = a \pm i\beta$, the correct provides $\lambda_1, \lambda_2 = a \pm i\beta$, the correct	 A₀, A₁ A₁ A₁ A₁ A₁ A₁ A₂ A₁ A₂ A₁ A₁ A₂ A₁ A₂ A₁ A₂ A₁ A₂ A₁ A₂ A₁ A₁ A₁ A₁ A₁ A₁ A₁ A₁ A₁ 	(i), $Q_i(t), A_i(t), B_i(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$ $\zeta_i \subset D \in \mathbb{R}$ $\exists_i \in \mathbb{Q}_0 = \mathbb{R}$ both terms mus- term or terms of y_i is found in the duplication. Variation of Parameter heady variation of param- tic duplication of param- terms of y_i is found in the theorem of the duplication. Variation of Parameter Interpreting Non-Real Eigen $e^{-d} = \begin{bmatrix} \cos(\beta_i) & -\sin(\beta_i) \\ (\beta_i) + \cos(\beta_i) \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix}$ instructional bediefine the equation of the duplication of the duplicat	t be in y_0 even if only one term is y_0 , multiply the term by t or t^2 to S sets to find the solutions of $y' +$ applied to second order equations ps. nvalues dom. und. assification uilibrium solution. An equilibrium nt. o t as $t \to \infty$ we call this neutrally or repelled, we call this neutrally unster Plane
• If $m < n$ 12.4.1 Wro The Wronski tions. This Wronski tions. This Wronski tions. This Wronski $L(y) = a_{c}'$ (), independent 12.5 Uro Let's assume $t \in$ some inter • Equilibri • Equilibri • Equilibri • Speed is 14.2 Urin Let's assume $T \in S$ and $T \in T$ • Equilibri • Speed is 14.2 Urin • End size • Find size • Find size • Solution: 14.3 Nou If we have a sy 25. Find size • Solution: 14.3 Nou If we have a sy 26. Solution: 14.3 Nou If we have a sy 27. Solution: 14.3 Nou If we have a sy 27. Solution: 14.3 Nou If so the save a sy 27. Solution: 14.3 Nou If so the save a sy 37. Solution: 15. Find size • Construct 37. Solution: 15. Solution: 16. Solution: 17.	the set does not span the omkin an also tells on about the li formkan is identical to the p_1, p_1, p_2, p_3 by a set of sub- formkan is identical to the p_1, p_2, p_3 by p_3 and p_4 by the set of the	space. inear independence of a set of funce Wronskian previously defined (??) itions of an nth order homogeneous $a_i(t)y' + a_i(t)y = 0$ it on (a, b) , then the set is linearly ents ents ents entric) of the eigenvalues. ear Eigenvalues. ear Eigenvalues. entric = 0 entric = 0 entri	• A ₀ , k_{xx} , k_{xy} , k_{yy}	(i), $Q_i(t), A_i(t), B_i(t) \in \mathbb{P}$ $B_0 \in \mathbb{P}_0 = \mathbb{R}$ $\zeta_i \subset D \in \mathbb{R}$ $\exists_i \in \mathbb{P}_0 = \mathbb{R}$ both terms mus- term or terms of y_i is found in the duplication. Variation of Parameter heady variation of param- tic duplication of param- term of $(0, j = q_i)$ and $(0, j = q_i)$ $(0, j = q_i) = (1, j = q_i)$ in the duplication of param- $(0, j = q_i) = (1, j = q_i)$ in the duplication of param- $(1, j = q_i) = (1, j = q_i)$ in the duplication of param- $(1, j = q_i) = (1, j = q_i)$ in the duplication of param- $(1, j = q_i) = (1, j = q_i)$ in the duplication of param- $(1, j = q_i) = (1, j = q_i)$ in the duplication of $g > 0$ $(1, j = q_i) = (1, j = q_i)$ is then difficus tilt and shape. Stability and Linear CI and bottoms are neither attracted in the parameter Plane Parameter Plane Parameter Plane Parameter Eigenvalue , $(1 < q_i) > 0$ $(1 > q_i) = (1 < q_i) > 0$	t be in y_0 even if only one term is y_0 , multiply the term by t or t^2 to S sters to find the solutions of $y' +$ applied to second order equations ps. nvalues dom. und. assification uilbrium solution. An equilibrium nt. o t as $t \to \infty$ we call this neutrally or repelled, we call this neutrally interer Plane assibilities.

12 Higher Order Linear Differential Equa- tions $m\bar{x} + b\dot{x} + kx = f(t)$ (23)	the methods given ahead, be sure to come back and determine how the solutions were determined. Given Equation: $m\ddot{x} + kx = 0$ $x(t) = c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t)$
	$\omega_0 = \sqrt{\frac{k}{m}}$
12.1 Harmonic Oscillators 12.1.1 The Mass-Spring System	This gives us one form of the solution, however we can also find an al nate form:
Consider an object with mass m on a table that is attached to a spring	$x(t) = A \cos(\omega_0 t - \delta)$
attached to wall. When the object is moved by an external force, we can	Where • Amplitude A and phase angle δ (radians) are arbitrary constants de
model its behavior using Newton's Second Law of Motion: $F = m\ddot{x}$ where F is the sum of the forces acting on the object.	 Amplitude A and plase angle 6 (ratians) are arbitrary constants de mined by initial conditions.
We have three different types of forces:	• The motion has circular frequency $\omega_0 = \sqrt{\frac{k}{m}}$ (radians) per second,
- Restoring Force: The restorative force of a spring is \propto the amount of stretching/compression: $F_{restoring} = -kx$	a natural frequency $f_0 = \frac{m_0}{2\pi}$
Damping Force: We also assume that friction exists and therefore	 The period T (seconds) is 2π√^m/_k
a damping force \propto the velocity of the object: $F_{damping} = -b\dot{x}$ Where	 The above solution is a horizontal shift of A cos(ω₀t) with phase s δ/.
damping constant $b > 0$ and small for slick surfaces.	To convert between the two forms, apply the following formulas.
 External Force: We also allow for an external force to drive the motion: F_{external} = f(t) 	$\begin{cases} A = \sqrt{c_1^2 + c_2^2} \\ s = \delta \\ s =$
Thus we get our equation for a Simple Harmonic Oscillator:	$\begin{cases} \tan \delta = \frac{i\alpha}{c_1} \\ \text{To solve the Mass-Spring System with both damping and forcing as give the following contaction:} \end{cases}$
$m\ddot{x} + b\dot{x} + kx = f(t)$	by the following equation: $m\ddot{x} + b\dot{x} + kx = F_0 \cos(\omega t)$
 Constants m > 0, k > 0, b > 0 	we can apply the following formula. (Note, some concepts are explain
- When $b=0,$ the motion is called undamped. Otherwise it is damped.	later in the text, refer back if needed) 1. $x_k(t)$ has three possible solutions. See (??).
• if $f(t) = 0$, the equation is homogeneous and the motion is called	 x_p(t) has the possible stational dec (11). x_p(t) can be assumed as A cos(ω_ft) + B sin(ω_ft) See (??).
unforced, undriven, or free. Otherwise it is forced, or driven.	
12.1.2 Solutions	3. $\omega_0 = \sqrt{\frac{k}{m}}$
When we say solution, we are referring to a solution that gives us x , in other	4. $A = \frac{m(\omega_0^2 - \omega_l^2)F_0}{m^2(\omega_0^2 - \omega_l^2)^2 + (b\omega_l)^2}$
words, the position of the mass at any given time t as a function of t . Due to the inherent nature of derivatives, this may or may not have undetermined	5. $B = \frac{k\omega/F_0}{m^2(\omega_0^2 - \omega_0^2)^2 + (k\omega/)^2}$
constants (often denoted as $[c_1, c_2,, c_n]$) as will be set by initial values given (similar to first order differential equations).	As you can see, this is a pain. Values A and B in particular are tedi
Later we will determine how to solve these equations fully, however a quick	to calculate. Despite this, as you'll see later, these methods can be ea
	than solving by hand.
 Find two linearly independent solutions of the second order differential 1 equation y" + p(t)y' + q(t)y = f(t) this having the general solution 	
equation $y' + p(t)y' + q(t)y' = f(t)$ this naving the general solution $y_h(t) = c_1y_1(t) + c_2y_2(t)$	
2. To find the particular solution, take $y_h(t) = c_1y_1(t) + c_2y_2(t)$ and swap constants to get $u_1(t) = u_2(t)u_1(t) + u_2(t)u_1(t)$ where u_1 and u_2 are	1. Write the characteristic equation $ A - \lambda I = 0$
constants to get $y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$ where v_1 and v_2 are unknown functions.	Solve the characteristic equation for the eigenvalues.
3. We find $v_1 \mbox{ and } v_2$ by substituting our new equation into our first. Dif-	3. For each eigenvalue, find the eigenvector by solving $(A-\lambda_i I)\vec{\mathbf{v}}_i=$
ferentiating by the product rule we get $y_p^\prime(t)=v_1y_1^\prime+v_2y_2^\prime+v_1^\prime y_1+v_2^\prime y_2$	As you'd imagine, once the size of a matrix becomes larger than 2 of
 Before we calculate y^p_p we choose an auxiliary condition, that v₁ and v₂ satisfy vⁱ₁y₁ + vⁱ₂y₂ = 0 where we get y^p_p = v₁yⁱ₁ + yⁱ₂v₂ 	these steps are tedious and long. Computers to the rescue!
	13.1 Special Cases
6. We wish to get $L(y) = y'' + py' + qy = f$ Substituting for what we have	-
solved for gives $v_1y'_1 + v'_2y'_2 = 0$	 Triangular Matrices: The eigenvalues of a triangular matrix (up
7. We now have two equations for our two unknowns. $\begin{cases} y_1v'_1 + y_2v'_2 = 0 \\ y'_1v'_1 + y'_2v'_2 = f \end{cases}$	 Inaugural Matrices. The eigenvalues of a triangular matrix (up or lower) appear on the main diagonal.
	+ 2 \times 2 Matrices: The eigenvalues can be determined with λ
Solve the system of equations and insert.	$(Tr^4(\mathbf{A}))\lambda + \mathbf{A} = 0$
Another method is to use Cramer's Rule (??) where $\begin{bmatrix} 0 & y_2 \end{bmatrix}$ $\begin{bmatrix} y_1 & 0 \end{bmatrix}$	 3 × 3 Matrices: Similarly: λ³ - λ²Tr(A) - λ¹/₂ (Tr(A²) - Tr²(A) det(A) = 0
$v'_{1} = \frac{ f - y'_{2} }{ f - y'_{2} }$ and $v'_{1} = \frac{ y'_{1} - f }{ f - f }$	uc(x) = 0
$y_1 y_2 = y_1 $	13.2 Eigenspaces
The denominator in this case is the Wronskian. It will not be zero because	The set of all eigenvectors belonging to an eigenvalues λ together with zero vector form a subspace of \mathbb{R}^n called the eigenspace.
	Theorem 7 (Eigenspaces). For each eigenvalue λ of a linear transformat $T : \mathbb{V} \to \mathbb{V}$, the eigenspace $\mathbb{E}_{\lambda} = \{\vec{V} \in \mathbb{V} T(\vec{v}) = \lambda \vec{v}\}$ is a subspace of \mathbb{V}
Vectors that aren't rotated by linear transformations, but are only scaled or flipped are called eigenvectors.	Theorem 8 (Distinct Eigenvalue). Let A be an $n \times n$ matrix.
mpped are caned eigenvectors. Theorem 6 (Eigenvalues and Eigenvectors). Let $T : V \rightarrow V$ be a linear	$\lambda_1, \lambda_2, \dots, \lambda_p$ are distinct eigenvalues with corresponding eigenvect $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$, then $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is a set of linearly independent vect
transformation. A scalar λ is an eigenvalue of T is there is a nonzero vector	In other words, if each eigenvalue has one associated eigenvector, than
$\vec{v} \in V$ such that $T(\vec{v}) = \lambda \vec{v}$.	
	set of eigenvectors is linearly independent.
Such a nonzero vector \vec{v} is called an eigenvector of T corresponding to λ . If the linear transformation T is regenerated by an $n \times n$ matrix A where	³ Note, the same exact steps are followed even if we have λ to be in terms of <i>i</i> , only eccention is that we are no longer in any \mathbb{R}^n space and therefore there will be
Such a nonzero vector \vec{v} is called an eigenvector of T corresponding to λ . If the linear transformation T is regenerated by an $n \times n$ matrix A where	³ Note, the same exact steps are followed even if we have λ to be in terms of <i>i</i> . only eccention is that we are no longer in any \mathbb{R}^n space, and therefore there will be
Such a nonzero vector \vec{v} is called an eigenvector of T corresponding to λ . If the linear transformation T is regenerated by an $n \times n$ matrix A where $\forall = \mathbb{R}^n$ and $T(\vec{v}) = A\vec{v}$, then A and \vec{v} are characterized by the equation $A\vec{v} = \lambda \vec{v}$.	³ Note, the same exact steps are followed even if we have λ to be in terms of <i>i</i> . only exception is that we are no longer in any \mathbb{R}^n space, and therefore there will be real eigenspace (Sec [77]) ⁴ Where $Tr(\mathbf{A})$ is the Trace of a matrix, i.e. the sum of the main diagonal.
Such a nonzero vector \vec{v} is called an eigenvector of T corresponding to λ . If the linear transformation T is regenerated by an $n \times n$ matrix A where	⁻¹ Note, the same exact steps are followed even if we have λ to be in terms of i. only exception is the so longer in any R ⁿ space, and therefore there will be obligation of experiment (see (77)). ⁻¹ Where T(A) is the Trace of a matrix, i.e. the sum of the main diagonal. ⁻¹ Where T(A) is the Trace of a matrix, i.e. the sum of the main diagonal. ⁻¹ Where T(A) is the Trace of a matrix, i.e. the sum of the main diagonal. ⁻¹ Where T(A) is the Trace of a matrix. ⁻¹ Where T(A) is the Trace of a matrix.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\nu \times T$ matrix A sheers $\Psi = \mathbb{R}^{4}$, and $T(\bar{\mathbf{v}}) = A\bar{\mathbf{v}}$, then A and $\bar{\mathbf{v}}$ are characterized by the equation $A\Phi = \lambda \Phi$. The signs of the eigenvalues direct the trajectory behavior in the phase- l-i portrait. We can label the eigendirections fast or slow based on the magnitude	Twist, the same start steps are followed even if we have \label{eq:steps} to be the wave loss parts are and \$
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\times \tau$ matrix A where $\Psi = \mathbb{R}^n$ and $T(\bar{\Psi}) = A\bar{\Psi}$, then A and $\bar{\Psi}$ are characterized by the equation $A\Psi = A\bar{\Psi}$. The signs of the eigenvalues direct the trajectory behavior in the phasel- portrait.	⁻¹ Note, the same exact steps are followed even if we have λ to be in terms of i. only exception is the so longer in any R ⁿ space, and therefore there will be obligation of experiment (see (77)). ⁻¹ Where T(A) is the Trace of a matrix, i.e. the sum of the main diagonal. ⁻¹ Where T(A) is the Trace of a matrix, i.e. the sum of the main diagonal. ⁻¹ Where T(A) is the Trace of a matrix, i.e. the sum of the main diagonal. ⁻¹ Where T(A) is the Trace of a matrix. ⁻¹ Where T(A) is the Trace of a matrix.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $n \times n$ matrix A sheer $\Psi = g^{\alpha}$ and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation M = Ac. The signs of the eigenvalues direct the trajectory behavior in the phase- 1 portrait. We can label the eigendrections fast or slow based on the magnitude of the eigenvalues. Whichever it is, the trajectorize are parallel to fast	¹ Note, the same exact steps are followed even if we have λ to be in terms of t, and overprofile in that we are so larger in an WP space, and therefore there will be vale space (See (TI)). Where T(A) is the line of a matrix, i.e. the sum of the main diagonal. 4 (b) Star Node: IA has two linearly independent eigenvectors we it an attracting or repelling star mode. The sign of A gives stahility. In both cases, the sign of A gives its stahility. In both cases, the sign of A gives its stahility.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\lambda \sim n$ matrix A shows $\Psi = \mathbb{R}^d$, and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $M = -Nc$. The signs of the eigenvalues direct the trajectory behavior in the phase-1-portant. We can label the eigenductorises fast or slow based on the magnitude of the eigenvalues. Whichever it is, the trajectories are parallel to fast and prependicular to slow. Three possibilities • Attracting Node $(\lambda_i < \lambda_2 < 0)$	 ^{¬Notes} the same exact stops are followed even if we have λ to be in terms of t, and opperception is that we are to longer in an BP ² paper, and therefore there will be real eigenvectors. (See (71)) ^{NUMMET} T/A) is but frame of a matrix, i.e. the sum of the main diagonal. (4) Star Node: If λ has two linearly independent eigenvectors we is an attracting or repelling star node. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. In hoth cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity, parallel to \$\mathcal{V}\$.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\nu \times \tau$ matrix A shere $\Psi = \mathbb{R}^{4}$, and $T(\nabla) = A\nabla$, then A and ∇ are characterized by the equation $A\nabla = \lambda E$. The signs of the eigenvalues direct the trajectory behavior in the phasel- portrait. We can label the eigenvalues direct the trajectories are parallel to fast and prependentiant to slow. Three possibilities • Attracting Node $(\lambda < \lambda_{2} < 0)$ • Repetiting Node $(0 < \lambda_{1} < \lambda_{2})$	¹ Note, the same exact steps are followed even if we have λ to be in terms of t, and operoption is that we are so larger in an WP space, and therefore there will be vale space (See (TI)). Where T(A) is the line of a matrix, i.e. the sum of the main diagonal. 4 (b) Star Node: IA has two linearly independent eigenvectors we it an attracting or repelling star mode. The sign of A gives stahility. In both cases, the sign of A gives its stability.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\nu \times \tau$ matrix A sheers $\Psi = \mathbb{R}^{4}$, and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\Phi = \lambda \Phi$. The signs of the eigenvalues direct the trajectory behavior in the phasel- portrait. We can label the eigenvalues direct the trajectories are parallel to fast and prependicular to show. Three possibilities • Attracting Node $(\lambda_{1} < \lambda_{2} < 0)$ • Repetiling Node $(0 < \lambda_{1} < \lambda_{2})$	 ^{¬Notes} the same start steps are followed even if we have λ to be in terms of t, and y coreption is that we are to larger in an 9% rapse, and therefore there will be real eigenvalues (See (71)). ^{NMEWF} 7/A) show there of a matrix, i.e. the sum of the main diagonal. (4) Star Note: IA has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of A gives stahilty. In both cases, the sign of A gives its stability. In both cases, the sign of A gives its stability. If λ > 0, trajectories go to infinity parallel to v? If λ < 0, trajectories go to finitely parallel to v?
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\lambda \sim n$ matrix A shows $\Psi = \mathcal{K}^{\alpha}$ and $T(\tilde{\nabla}) = A\tilde{\nabla}_{1}$ then A and $\tilde{\nabla}$ are characterized by the equation $M^{\alpha} = Nc$. The signs of the eigenvalues direct the trajectory behavior in the phasel-i- portant: We can label the eigenvalues direct the trajectory behavior in the phasel-i- gentration of the eigenvalues. Which ever it is, the trajectories are parallel to fast and perpendicular to show. Three possibilities • Attracting Node $(\lambda_{1} < \lambda_{2} < 0)$ • Repetiting Node $(\lambda_{1} < \lambda_{2} < 0)$ • Studile Point $(\lambda_{1} < 0 < \lambda_{1})$ • Complex Conjugate Eigenvalues $(\Delta < 0)$	 ^{¬Notes} the same start steps are followed even if we have λ to be in terms of t, and y everytion is that we are to longer in an BP ² paper, and therefore there will be read expansion. Set (71) ^{NUMEP} T/A) is but frame of a matrix, i.e. the sum of the main diagonal. ⁴(4). Stare Yode: It λ has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity, parallel to 4[°], If λ > 0, trajectories go to finding, parallel to 4[°]. If λ = 0, there exists a line of fixed points at the eigenvector. 15 Non-Linear Systems
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\nu \times \tau$ matrix A shere $\Psi = \mathbb{R}^{4}$, and $T(\nabla) = A\nabla_{i}$, then A and ∇ are characterized by the equation $A\nabla = \lambda k$. The signs of the eigenvalues direct the trajectory behavior in the phasel- i portrait. We can label the eigenvalues direct the trajectories are parallel to fast and prependicular to slow. Three possibilities • Attracting Node $(\lambda_{i} < \lambda_{2} < 0)$ • Repelling Node $(0 < \lambda_{i} < \lambda_{2})$ • Saddle Point $(\lambda_{i} < 0 < \lambda_{2})$ • Saddle Point $(\lambda_{i} < 0 < \lambda_{2})$ • Complex Conjugate Eigenvalues $(\Delta < 0)$ When $\Delta = (TA)^{2} - (1A)^{2} < 0$ we get non-real eigenvalues.	 ^{¬Note} the same east steps are followed even if we have λ to be in terms of t, orgoteroptic ta have are no longer in any R² space, and therefore there will braid eigenvectors. (See (71)) ⁽⁴⁾ Where 77(A) is the Ther of a nutrit, i.e. the sum of the main diagonal. ⁽⁴⁾ (b) Star Node: If λ has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of λ gives stability. In both cases, the sign of λ gives its stability. If h > 0, trajectorize gap to a the orgin parallel to <i>Φ</i>. If λ > 0, trajectorize gap to the orgin parallel to <i>Φ</i>. If λ = 0, there exists a line of fixed points at the eigenvector. 15 Non-Linear Systems 15.1 Properties of Phase Plane Trajectories in N
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\lambda \sim n$ matrix A shows $\Psi = \mathbb{R}^{d}$, and $T(\nabla) = A\nabla_{i}$, then A and ∇ are characterized by the equation $A\nabla = \lambda A$. The signs of the eigenvalues direct the trajectory behavior in the phasel- i portrait. We can label the eigenvalues direct the trajectories are parallel to fast and prependentiatr to show. Three possibilities • Attracting Node $(\lambda < \lambda_{2} < 0)$ • Repelling Node $(0 < \lambda_{1} < \lambda_{2})$ • Saddle Point $(\lambda_{1} < 0 < \lambda_{2})$ • Saddle Point $(\lambda_{1} < 0 < \lambda_{2})$ • Complex Conjugate Eigenvalues $(\Delta < 0)$ When $\Delta = (TA)^{2} - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \pm \beta i$ where $\alpha = \frac{\Psi(\alpha)}{2}$ and $\beta = \sqrt{-\Delta}$, α and β are real. The real solutions	 ^{¬Notes} the same start steps are followed even if we have λ to be in terms of t, and y everytion is that we are to longer in an BP ² paper, and therefore there will be read expansion. Set (71) ^{NUMEP} T/A) is but frame of a matrix, i.e. the sum of the main diagonal. ⁴(4). Stare Yode: It λ has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity, parallel to 4[°], If λ > 0, trajectories go to finding, parallel to 4[°]. If λ = 0, there exists a line of fixed points at the eigenvector. 15 Non-Linear Systems
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\lambda \sim n$ matrix A shows $\Psi = R^{\alpha}$. The signs of the eigenvalues direct the trajectory behavior in the phase- 1_{2} portrait. We can label the eigenvalues direct the trajectory behavior in the phase- 1_{2} portrait. We can label the eigenvalues direct the trajectory behavior in the phase- 1_{2} of the eigenvalues. Which ever it is, the trajectories are parallel to fast and prependicular to show ($\lambda_{1} < \lambda_{2} < 0$) • Repelling Node ($\lambda_{1} < \lambda_{2} < 0$) • Repelling Node ($\lambda_{2} < \lambda_{2} < 0$) • Sindle Potent ($\lambda_{1} < 0 < \lambda_{2} < \lambda_{2}$) • Sindle Potent ($\lambda_{1} < 0 < \lambda_{2} < 0$) • Sindle Potent ($\lambda_{1} < 0 < \lambda_{2} < 0$) When $\Delta_{2} = (T_{1}/T_{1})^{2} - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha + \frac{1}{2}$ in β where $\alpha = \frac{10}{2}$ in β $\sqrt{-\Delta}$. α and β are real. The real solutions are given by:	 ^{¬Notes} the same sets togen are followed even if we have λ to be in terms of t. and opportuping the have are to longer in an B² page, and therefore there will be real eigenvectors. Set (71) ^{NOTE} T/A) is but frace of a matrix, i.e. the sam of the main diagonal. ⁴(4). Star Node: It λ has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity, parallel to σ². If λ > 0, trajectories go to finder points at the eigenvector. 15 Non-Linear Systems 15.1 Properties of Phase Plane Trajectories in Ne Linear 2 × 2 Systems When uniqueness holds, phase plane trajectories cannot cross.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\lambda \sim n$ matrix A shows $\Psi = \mathbb{R}^n$ and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\bar{\nabla} = A\bar{\nabla}$. The signs of the eigenvalues direct the trajectory behavior in the phasel- portrait. We can label the eigenvalues direct the trajectories are parallel to fast and prependentials to slow. Three possibilities • Attracting Node $(1 < \lambda_3 < 0)$ • Repedling Node $(0 < \lambda_1 < \lambda_2)$ • Saddle Point $(\lambda_1 < 0 < \lambda_2)$ • Saddle Point $(\lambda_1 < 0 < \lambda_2)$ • Complex Conjugate Eigenvalues $(\Delta < 0)$ When $\Delta = (Tr(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \pm \beta i$ where $\alpha = \frac{P_{1,2}}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\left\{ \mathbf{x}_{n} = \mathbf{e}^{-i}(\cos(\beta)) \mathbf{\beta} - \sin(\beta) \mathbf{q} \right\}$	 ^{¬Notes} the same sets togen are followed even if we have λ to be in terms of t. and opportuping the have are to longer in an B² page, and therefore there will be real eigenvectors. Set (71) ^{NOTE} T/A) is but frace of a matrix, i.e. the sam of the main diagonal. ⁴(4). Star Node: It λ has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity, parallel to σ². If λ > 0, trajectories go to finder points at the eigenvector. 15 Non-Linear Systems 15.1 Properties of Phase Plane Trajectories in Ne Linear 2 × 2 Systems When uniqueness holds, phase plane trajectories cannot cross.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $n \times n$ matrix A sheer $\Psi = \mathbb{R}^n$, and $T(\nabla) = A\nabla$, then A and \bar{Y} are characterized by the equation $A\nabla = AV$. The signs of the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectories are parallel to fast and perpendicular to skew. Three possibilities • Attracting Node $(\lambda_1 < \lambda_2 < 0)$ • Repelling Node $(0 < \lambda_1 < \lambda_2)$ • Sadile Poult $(\lambda_1 < 0 < \lambda_2 < 0)$ • Repelling Node $(0 < \lambda_1 < \lambda_2)$ • Sadile Poult $(\lambda_1 < 0 < \lambda_2 < 0)$ When $\Delta = (\Gamma(A_1)^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \pm \frac{\beta}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\left\{ \vec{x}_1 = e^{-i(\alpha_1(B_1)\vec{P} - i\alpha_1(B_1)\vec{Q})} \\ \vec{x}_2 = e^{-i(\alpha_1(B_1)\vec{P} - i\alpha_1(B_1)\vec{Q})} \\ \end{array} \right\}$	 ^{¬Notes} the same sets steps are followed over if we have λ to be in term of ℓ. ^{Notes} r(r, k) are we to larger an an #P space, and therefore there will be not experiment (see (T1)). ^{NOTE} r(r, k) are there of a matrix, i.e. the sum of the main diagonal. ⁽⁴⁾ (b) Star Note: It λ has two linearly independent eigenvectors we it an attracting or repulling star node. The sign of λ gives stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ = 0, there exists a line of fixed points at the eigenvector. IS 1. Properties of Phase Phane Trajectories cannot cross. When the given function A and a are continuous trajectories are complexed to a continuous trajectories are complexed.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\lambda \sim 0$ matrix A shows $\Psi = \mathbb{R}^n$ and $T(\bar{\nu}) = A\bar{\nu}$, then A and $\bar{\nu}$ are characterized by the equation $M = A\bar{\nu}$. The signs of the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectory behavior in the phase4 portrait. Three possibilities • Attracting Node $(\lambda_1 < \lambda_2 < 0)$ • Repetiting Node $(\lambda_1 < \lambda_2 < 0)$ • Repetiting Node $(\lambda_1 < \lambda_2 < 0)$ • Roughext ($\lambda_1 < 0 < \lambda_2 < 0$) • Roughext ($\lambda_1 < 0 < \lambda_2 < 0$) • Roughext ($\lambda_1 < 0 < \lambda_2 < 0$) • Roughext ($\lambda_1 < 0 < \lambda_2 < 0$) • Roughext ($\lambda_1 < 0 < \lambda_2 < 0$) • Roughext ($\lambda_1 < 0 < \lambda_2 < 0$) • Repetiting $A = (T_1A)^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \pm \beta \beta$ where $\alpha = \frac{T_1A}{2}$ and $\beta = \sqrt{-\Delta}$. Λ and β are real. The real solutions are given by: $\left\{ \vec{\lambda}_1 = -\alpha(\cos(\beta)\beta - \sin(\beta)) + \alpha(\beta) \right\}$ For complex eigenvalues stability behavior depends on the sign of α . • Attractive Solution ($\Delta = 0$)	 ^{¬Notes} the same set steps are followed even if we have λ to be in terms of t, and composite in the are so long to an a PF space, and therefore there will be not expense that there is the set of the same of the main digense. (See (71)) ^{NOTE} (74) All there is the set of matrix, i.e. the sum of the main digense. (See (71)) ^{NOTE} (74) All there is the set of matrix, i.e. the sum of the main digense. (See (74)) ^{NOTE} (74) All there is the set of matrix is the sum of the main digense. ^{III} λ > 0, trajectories go to infinity, parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} λ > 0, trajectories approach the origin parallel to ₹. ^{III} N = 0, there exists a line of fixed points at the eigenvector. ^{III} N = 0, there fixed of Phase Phase Phase The origin to approach or the origin of Phase Phase Phase trajectories cannot cross. ^{III} When the given functions f and g are continuous, trajectories are trainous and smooth. ^{III} C C Equilibria Phase Phase trajectories to approach we can be made than one, or more at all. To find a system
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ -matrix A shows $\Psi = \mathbb{R}^n$ and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\bar{\nabla} = A\bar{\nabla}$. The signs of the eigenvalues direct the trajectory behavior in the phasel-iperturit. We can label the eigenvalues direct the trajectory behavior in the phasel-iperturit. We can label the eigenvalues direct the trajectories are parallel to fast and prependentiate to show. Three possibilities • Attracting Node $(\Lambda < \Lambda_X < 0)$ • Brayelling Node $(0 < \lambda_1 < \lambda_2)$ • Saddle Point $(\lambda_1 < 0 < \lambda_2)$ • Complex Conjugate Eigenvalues $(\Delta < 0)$ When $\Delta = (Tr(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \pm \beta \frac{1}{10}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\{ \mathbf{x}_{i} = e^{-ip}(\cos(\beta)) \mathbf{\beta} - \sin(\beta) \mathbf{\alpha} \}$ $\{ \mathbf{x}_{i} = e^{-ip}(\cos(\beta)) \mathbf{\beta} - \sin(\beta) \mathbf{\alpha} \}$ For complex eigenvalues sublity behavior depends on the sign of α . • Attracting Spiral $(\alpha < 0)$	 ^{¬Note}te the same set steps are followed even if we have λ to be in terms of t. and y compton in that we are hanger an any 8² page, and therefore there will be not experiment that there of a matrix, i.e. the same of the main diagonal. (4) Star Note: If λ has two lines of a matrix, i.e. the same of the main diagonal. (4) Star Note: If λ has two lines of a matrix, i.e. the same of the main diagonal. (4) Star Note: If λ has two lines of a matrix, i.e. the same of the main diagonal. (4) Star Note: If λ has two lines of a mode. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ < 0, trajectories go to finitivy, parallel to \$\vec{v}\$. If λ < 0, trajectories go to finitivy, parallel to \$\vec{v}\$. If λ = 0, there exists a line of fixed points at the eigenvector. 15. Non-Linear Systems 15.1 Properties of Phase Plane Trajectories in Nt Linear 2 × 2 Systems When uniqueness holds, phase plane trajectories cannot cross. When the given function ρ and g are continuous, trajectories are timous and amouth.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an $\kappa \times 0$ matrix A sheer $\Psi = \mathbb{R}^n$, and $T(\Psi) = A\Psi$, then A and Ψ are characterized by the equation $A\Psi = AV$. The signs of the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectories are parallel to fast and perpendicular to slow. Three possibilities • Attracting Node $(\lambda_1 < \lambda_2 < 0)$ • Repelling Node $(0 < \lambda_1 < \lambda_2)$ • Sadile Poult $(\lambda_1 < 0 < \lambda_2 < 0)$ • Repelling Node $(0 < \lambda_1 < \lambda_2)$ • Sadile Poult $(\lambda_1 < 0 < \lambda_2 < 0)$ When $\Delta = (\Gamma(A_1)^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \pm \frac{\beta}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\begin{cases} \xi_1 = -e^{-i(\alpha_1(\beta_1)\Phi} - i\alpha_1(\beta_1)Q) \\ \xi_2 = -e^{-i(\alpha_1(\beta_1)\Phi} - i\alpha_1(\beta_1)Q) \\ \xi_3 = -e^{-i(\alpha_1(\beta_1)\Phi} - i\alpha_1(\beta_1)Q) \\ K = complex cigravalues stability behavior depends on the sign of \alpha.$	 ^{¬Note} the same set steps are followed even if we have λ to be in terms of t, and composite in the are so long to an a PF space, and therefore there will be not expense that there is the set of an antity, i.e. the sum of the main diagonal. (4) Star Node: It A has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of λ gives stability. In both cases, the sign of λ gives its stability: If λ > 0, trajectories go to infinity, parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. Is 1. Properties of Phase Phane Trajectories cannot cross. Is the munipursues holds, phase phase trajectories cannot cross. 2. When the given functions <i>A</i> and <i>g</i> are continuous, trajectories are transon and smooth. Is Deaperties and have more than one, or none at all. To find a systee equilibria.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ - numbrix A shows $\Psi = \mathbb{R}^n$, and $T(\tilde{v}) = A\tilde{v}$, then A and \tilde{v} are characterized by the equation $A\tilde{v} = A\tilde{v}$. The signs of the eigenvalues direct the trajectory behavior in the phase- 1 portrait. We can label the eigenvalues direct the trajectory behavior in the phase- 1 portrait. We can label the eigenvalues direct the trajectory behavior in the phase- 1 portrait. We real help the eigenvalues fact or slow based on the magnitude of the eigenvalues. Whichever it is, the trajectories are parallel to fast and perpendicular to slow ($\lambda_i < \lambda_2 < 0$) Repelling Node $(0 < \lambda_i < \lambda_2)$ 2. Complex Conjugate Eigenvalues $(\Delta < 0)$ When $\Delta = (T(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = a \le \delta$; where $a = \frac{b(a)}{2}$ and $\beta = \sqrt{-\Delta}$. a and β are real. The real solutions are given by: $\left\{ \mathbf{x}_i = e^{-i(\mathbf{x})}(\beta)\mathbf{p} + -i\alpha_i(\beta)\mathbf{q} \right\}$ For complex eigenvalues stability behavior depends on the sign of α . A tracting Spiral $(\alpha < 0)$ Repelling Spiral $(\alpha < 0)$	 ^{¬Note} the same sear tops are followed even if we have λ to be in terms of t. and y composite in the torms of a matrix, i.e. the same of the main degenetation (See (71)). ^{NOTE} (74) absorb Theore of a matrix, i.e. the same of the main dagonal. ⁽⁴⁾ (1) Star Note: It λ has two linearly independent eigenvectors we it an arterating or repelling star mode. The sign of λ gives its stability. ⁽⁴⁾ In both cases, the sign of λ gives its stability. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ When the given functions f and g are continuous, trajectories are tumous and smooth. ⁽⁴⁾ Executive that have than one, or none at all. To find a syste equilibra side: <i>and g</i> y simultaneously. ⁽⁵⁾ S Nullclines
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ - numbrix A show: $\Psi = \mathbb{R}^n$ and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\bar{\Psi} = A\bar{\nu}$. The signs of the eigenvalues direct the trajectory behavior in the phasel-ip- portrait. We can label the eigenvalues direct the trajectory behavior in the phasel-ip- portrait. We can label the eigenvalues direct the trajectory behavior in the phasel-ip- portrait. We can label the eigenvalues fact or alow based on the magnitude of the eigenvalues. Whichever, it is, the trajectories are parallel to fast and perpendicular to slow ($\lambda_1 < \lambda_2 < 0$) • Repeting Node ($0 < \lambda_1 < \lambda_2$) • Sodule Point ($\lambda_1 < 0 < \lambda_2$) • Sodule Point ($\lambda_1 < 0 < \lambda_2$) • Complex Conjugate Eigenvalues ($\Delta < 0$) When $\Delta = (T(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \neq i$ where $\alpha = \frac{b(\psi)}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: E or complex eigenvalues ($\alpha < 0$) • Repeting Spiral ($\alpha < 0$) • Repeting Spiral ($\alpha < 0$) • Contract ($\alpha = 0$) 3. Borderline Case: Zero Eigenvalues ($ A = 0$) If one eigenvalues is zero we get a now of non-isolated fixed points in the eigenvalues	 ^{¬Note} the same set steps are followed even if we have λ to be in terms of t, and composite in the are so long to an a PF space, and therefore there will be not expense that there is the set of an antity, i.e. the sum of the main diagonal. (4) Star Node: It A has two linearly independent eigenvectors we it an attracting or repelling star node. The sign of λ gives stability. In both cases, the sign of λ gives its stability: If λ > 0, trajectories go to infinity, parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. If λ > 0, trajectories approach the origin parallel to ₹. Is 1. Properties of Phase Phane Trajectories cannot cross. Is the munipursues holds, phase phase trajectories cannot cross. 2. When the given functions <i>A</i> and <i>g</i> are continuous, trajectories are transon and smooth. Is Deaperties and have more than one, or none at all. To find a systee equilibria.
Such a manzero sector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an κ - nutrix A shows $\Psi = \mathbb{R}^n$, and $T(\nabla) = A\nabla$, then A and \bar{Y} are characterized by the equation $A\nabla = AV$. The signs of the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectory behavior in the phase4 portrait. We can label the eigenvalues direct the trajectory behavior in the phase4 port ($A = AV$. Three possibilities • Attracting Node ($\lambda_1 < \lambda_2 < 0$) • Repelling Node ($0 < \lambda_1 < \lambda_2$) • Sadle Pout ($\lambda_1 < 0 < \lambda_2 < 0$) • Repelling Node ($0 < \lambda_1 < \lambda_2$) • Sadle Pout ($\lambda_1 < 0 < \lambda_2 < 0$) When $\Delta = (\Gamma(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \pm \frac{\beta}{24}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\left\{ \mathbf{x}_{i} = -c(i\alpha(iA))\Phi - i\alpha(iA))(\mathbf{x}_{i})$ For complex eigenvalues stability behavior depends on the sign of α . • Attracting Spiral ($\alpha < 0$) • Repelling Spiral ($\alpha < 0$)	 ^{¬Note} the same sear tops are followed even if we have λ to be in terms of t. and y composite in the torms of a matrix, i.e. the same of the main degenetation (See (71)). ^{NOTE} (74) absorb Theore of a matrix, i.e. the same of the main dagonal. ⁽⁴⁾ (1) Star Note: It λ has two linearly independent eigenvectors we it an arterating or repelling star mode. The sign of λ gives its stability. ⁽⁴⁾ In both cases, the sign of λ gives its stability. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ It λ = 0, there exists a line of fixed points at the eigenvector. ⁽⁴⁾ When the given functions f and g are continuous, trajectories are tumous and smooth. ⁽⁴⁾ Executive that have than one, or none at all. To find a syste equilibra side: <i>and g</i> y simultaneously. ⁽⁵⁾ S Nullclines
Such a manzero sector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an κ - numbrix A shows $\Psi = \mathbb{R}^n$, and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\bar{\nabla} = A\bar{\nabla}$. The signs of the eigenvalues direct the trajectory behavior in the phase- \bar{U}_{p} portrait. We can label the eigendirections fact or slow based on the magnitude of the eigenvalues. Whichever it is, the trajectories are parallel to fast and prependicular to slow. Three possibilities • Attracting Node $(\lambda_i < \lambda_j < 0)$ • Rogelling Node $(0 < \lambda_i < \lambda_j)$ • Sodile Pout $(\lambda_i < 0 < \lambda_i)$ • Sodile Pout $(\lambda_i < 0 < \lambda_i)$ • Complex Conjugate Eigenvalues $(\Delta < 0)$ When $\Delta = (T(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{i,2} = a \pm \frac{1}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\left\{ S_{i,i} = e^{-i(\cos(\beta))} \beta = -i(\alpha_i)^2 \beta_{i,j} \right\}$ For complex eigenvalues stability behavior depends on the sign of α . • Attracting Spiral $(\alpha < 0)$ • Recreding Spiral $(\alpha < 0)$ • Recreding Case: Zezo Eigenvalues $(A = 0)$ If one eigenvalues masociated with the eigenvalues and the phase phase trajectories are all straight lines in direction of other eigenvector. Along which It wo eigenvalues are zen, there is only one eigenvector, along which is proved by the order of the eigenvalues (A = 0) If one eigenvalues masociated with the eigenvalues are zen, there is only one eigenvector, along which is proved by the order of the eigenvalues are all trajectories are all straight lines in direction of other eigenvector.	 ^{¬Note} the same set steps are followed even if we have λ to be in terms of t, and y composite in that use are to larger an any \$P space, and therefore there will be not experiment that there are a matrix, i.e. the same of the main diagonal. ⁽⁴⁾ (b) Sarr Note: It λ have two linearly independent eigenvectors we it an attracting or repelling star mode. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity parallel to \$\mathcal{P}\$. If λ > 0, trajectories approach the origin parallel to \$\mathcal{P}\$. If λ > 0, trajectories approach the origin parallel to \$\mathcal{P}\$. If λ > 0, there exists a line of fixed points at the eigenvector. 15. Non-Linear Systems 10. When the given functions <i>f</i> and <i>g</i> are continuous, trajectories are timous and smooth. 15.2 Equilibria Phase Potentiat: on them more than one, or none at all. To find a systeme of all simultaneously. 15.3 Nullclimes Nullclimes to also ead or the same as before. 15.4 Lineit Cycle A lined y elimina to a condition of a simultaneously.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ -matrix A show $\Psi = \mathbb{K}^n$ and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\bar{N} = A\bar{N}$. The signs of the eigenvalues direct the trajectory behavior in the phase-lipottania. Whichever it is, the trajectory behavior in the phase-lipottania. Whichever it is, the trajectories are parallel to fast and the eigenvalues. Whichever it is, the trajectories are parallel to fast and the eigenvalues. Whichever it is, the trajectories are parallel to fast and the eigenvalues. Whichever it is, the trajectories are parallel to fast and the eigenvalues. Whichever it is, the trajectories are parallel to fast and the eigenvalues. Whichever it is, the trajectories are parallel to fast and the eigenvalues. Whichever it is, the trajectories are parallel to fast and the eigenvalues. So that $\lambda_1 < 0 < \lambda_2$ is a state $\lambda_1 < 0 < \lambda_2$. Complex Conjugate Eigenvalues ($\Delta < 0$) When $\Delta = (T(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{1,2} = \alpha \neq 3i$, where $\alpha = \frac{W_1(A)}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\{\mathbf{x}_1 = -r(\cos(A))\hat{P} - \sin(A))\hat{Q}\}$ $\{\mathbf{x}_n = \alpha \in \mathbf{a}, $	 ^{¬Note} the same set steps are followed even if we have λ to be in terms of t, and y composite in that use are to larger an any \$P space, and therefore there will be not experiment that there are a matrix, i.e. the same of the main diagonal. ⁽⁴⁾ (b) Sarr Note: It λ have two linearly independent eigenvectors we it an attracting or repelling star mode. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity parallel to \$\mathcal{P}\$. If λ > 0, trajectories approach the origin parallel to \$\mathcal{P}\$. If λ > 0, trajectories approach the origin parallel to \$\mathcal{P}\$. If λ > 0, there exists a line of fixed points at the eigenvector. 15. Non-Linear Systems 10. When the given functions <i>f</i> and <i>g</i> are continuous, trajectories are timous and smooth. 15.2 Equilibria Phase Potentiat: on them more than one, or none at all. To find a systeme of all simultaneously. 15.3 Nullclimes Nullclimes to also ead or the same as before. 15.4 Lineit Cycle A lined y elimina to a condition of a simultaneously.
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ -matrix A show $\Psi = \mathbb{K}^n$ and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\bar{N} = A\bar{N}$. The signs of the eigenvalues direct the trajectory behavior in the phase-lipottania. In the signs of the eigenvalues direct the trajectory behavior in the phase-lipottania. The possibilities that the sign of the eigenvalues direct the trajectory behavior in the phase-lipottania. There possibilities • Attracting Node $(0 < \lambda_1 < \lambda_2 < 0)$ • Repetiting Node $(0 < \lambda_1 < \lambda_2)$ • Solidle Point $(\lambda_1 < 0 < \lambda_2)$ • Solidle Point $(\lambda_1 < 0 < \lambda_2)$ • Complex Conjugate Eigenvalues $(\Delta < 0)$ When $\Delta = (Tr(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{12} = \alpha \pm \frac{31}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\{ \vec{x}_1 = e^{-\alpha}(\cos(\beta)) \vec{p} - \sin(\beta)) \vec{q} \}$ $\{ \vec{x}_n = e^{-\alpha}(\cos(\beta)) \vec{p} - \sin(\beta)) \vec{q} \}$ $\{ \vec{x}_n = e^{-\alpha}(\cos(\beta)) \vec{p} - \sin(\beta)) \vec{q} \}$ For complex eigenvalues ($\beta < 0$) • Repeting Spiral $(\alpha < 0)$ • Repeting Spiral $(\alpha > 0)$ • Contract $\alpha = 0$ for an isolated fixed points in the eigenvalues are only an isolated fixed points in the eigenvalues are always are one consolated fixed points in the eigenvalue transformation is in the eigenvalue in the intervalue of the points. The toter is non-x one quevelocity is an eigenvalue in the eigenvalues in the eigenvalues in the eigenvalues in the eigenvalues in the eigenvalue in direction of other isolation in the eigenvalue in the point of the points. The toter is non-x one quevelocity is an eigenvalue in the eigenvalues in the eigenvalues in the eigenvalue in the eigenvalues in the eigenvalues in the eigenvalues in the eigenvalue in the eigenvalues in the ei	 ^{¬Note} the same set steps are followed even if we have λ to be in terms of i, and composite in the time for an end steps and the steps of a matrix, i.e. the sam of the main diagonal. ¹⁰(sigma) and the time of a matrix, i.e. the sam of the main diagonal. ¹⁰(sigma) and the same of the same steps are more than an end step of the same steps are more in an anternation or repulling star mode. The sign of λ gives stability. ¹¹ Is λ = 0, trajectories go to infinity, parallel to ₹. ¹¹ Is λ = 0, trajectories approach the origin parallel to ₹. ¹¹ Is λ = 0, trajectories approach the origin parallel to ₹. ¹¹ Is λ = 0, trajectories approach the origin parallel to ₹. ¹¹ Is λ = 0, trajectories approach the origin parallel to ₹. ¹¹ Is λ = 0, trajectories approach the origin parallel to ₹. ¹¹ Is λ = 0, trajectories approach the origin parallel to ₹. ¹¹ Is λ = 0, trajectories approach the origin parallel to ₹. ¹¹ Is λ = 0, there exists a line of fixed points at the eigenvectors. ¹² Non-Linear Systems ¹³ When uniqueness holds, phase phase trajectories cannot cross. ¹³ When the given functions <i>f</i> and <i>g</i> are continuous, trajectories are thranes and smooth. ¹⁵ Equilibrian ¹⁵ Nullelines ¹⁵ Nullelines ¹⁵ Nullelines ¹⁵ Nullelines ¹⁵ Nullelines in this case are the same a before. ¹⁵ L Limit Cycle ¹⁶ All solves that ones, or none at all. To find a systep end subtrast to the youthing around more and more codes from edite work from edite by working around more and more codes from the order solution in the youthing around more and more codes from the order solution in the youthing around more and more codes from the solution in the paralleline solution in the youthing around more and more code from c
Such a nonzero vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ - numbrix A shows $\Psi = \mathbb{R}^n$, and $T(\bar{\nabla}) = A\bar{\nabla}$, then A and $\bar{\nabla}$ are characterized by the equation $A\bar{\nabla} = A\bar{\nabla}$. The signs of the eigenvalues direct the trajectory behavior in the phasel-ip portrait. We can label the eigenvalues direct the trajectory behavior in the phasel-ip portrait. We can label the eigenvalues direct the trajectory behavior in the phasel-ip portrait. We can label the eigenvalues direct the trajectories are parallel to fast and prependicular to slow ($\lambda_i < \lambda_s < 0$) Bergelling Node ($0 < \lambda_i < \lambda_s$) Soluble Point ($\lambda_i < 0 < \lambda_i$) Soluble Point ($\lambda_i < 0 < \lambda_i$) Soluble Point ($\lambda_i < 0 < \lambda_i$) When $\Delta = (T(A))^2 - 4 A < 0$ we get non-real eigenvalues. $\lambda_{i,2} = \alpha \pm i$ where $\alpha = \frac{ib(i)}{2}$ and $\beta = \sqrt{-\Delta}$. α and β are real. The real solutions are given by: $\begin{cases} z_i = e^{\alpha i}(\cos(\beta t))\overline{\beta} - \sin(\beta t))$ E complex cignizability behavior depends on the sign of α . A transcring Spiral ($\alpha < 0$) Beroding Spiral ($\alpha < 0$) Beroding Spiral ($\alpha < 0$) Soluter in the eigenvalues ($A A = 0$) If one eigenvalues is zero we get a row of non-isolated fixed points in the eigenvalues is zero we get a row of non-isolated fixed points. The zerotes from any other here have a trave of non-isolated fixed points. The zerotes from any other here in the chase points. The zerotes from any other is zero we get now for non-isolated fixed points. The zerotes from any other here in particular bis from the sign points. The zerotes from any other here in particular bis constrained points. The zerotes from any other inversion specified by the system.	 ^{¬Note} the same set stops are followed over if we have λ to be in term of λ. ^{¬Note} repetite is the mesor of a matrix, i.e. the sam of the main degeneties there will be not experiment that there are no long are any PK space, and therefore there will be not experiment. Set (71) ⁽¹⁾ Star Note: If A has two linearly independent eigenvectors were it an attracting or repelling star node. The sign of A gives stability. In both cases, the sign of A gives its stability. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A > 0, trajectories approach the origin parallel to ℓ. If A = 0, there exists a line of fixed points at the eigenvector. Ison-Linear Systems When uniqueness holds, phase plane trajectories cannot cross. When the given functions / and g are continuous, trajectories are channes and smooth. Isons Portatios can have more than one, or none at all. To find a systee equilibria, solve a' and y' simultaneously. Lison Linear Cycles are are the same as before. Lison Linear Cycle is a closed curve (representing a periodic solution) to who here solutions terms hy wring a graving and more and more code from edit for solutions trajectories for a periodic solution in the solution terms in the solution terms in
Such a masses vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ - number λ shows $\Psi = \mathbb{R}^n$, and $T(\nabla) = A^n$, then A and Ψ are characterized by the equation $A^n = A^n$. The signs of the eigenvalues direct the trajectory behavior in the phased-portrait. We can hale the eigendirections fast or also based on the magnitude of the eigenvalues. Whichever it is, the trajectories are parallel to fast and perpendicular to slow. Three possibilities $-A \operatorname{tracting} Node (\lambda_1 < \lambda_2 < 0)$ $+ Repulling Node (0, < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Spirit (0, < 0)$ $+ \operatorname{Repulling Node (0, < $	 ^{¬Note} the same sets tops are followed over if we have λ to be in terms of ⊥. Only compute in the bar mere of a matrix, i.e. the same of the main degenetic field (2019). ^{Notest} T(A) also there of a matrix, i.e. the same of the main diagonal. ⁽⁴⁾ (4) Star Note: It λ has two linearly independent eigenvectors were it an anteracting or repelling star node. The sign of λ gives its stability. In both cases, the sign of λ gives its stability. If λ > 0, trajectories go to infinity, parallel to v?. If λ < 0, trajectories approach the origin parallel to v?. If λ < 0, trajectories approach the origin parallel to v?. If λ < 0, trajectories approach the origin parallel to v?. If λ < 0, there exists a line of fixed points at the eigenvectors. 15.1 Properties of Phase Plane Trajectories in Not Linear 2 × 2 Systems When turingeness holds, phase plane trajectories cannot cross. When the given functions f and g are continuous, trajectories are ot thmose and smooth. 15.2 Equilibria Phase Potratist can have more than one, or none at all. To find a syste equilibra, solve ≠ and g visualizaneously. 15.3 Nullclines Nullclines to chool curve frequencing a periodic solution) to whole solutions tend by winding around more and more closely from etimate or origin. 16 Linearization 16 Linearization
Such a masses vector Ψ is called an eigenvector of T corresponding to λ . If the linear transformation T is represented by an λ - number λ shows $\Psi = \mathbb{R}^n$, and $T(\nabla) = A^n$, then A and Ψ are characterized by the equation $A^n = A^n$. The signs of the eigenvalues direct the trajectory behavior in the phased-portrait. We can hale the eigendirections fast or also based on the magnitude of the eigenvalues. Whichever it is, the trajectories are parallel to fast and perpendicular to slow. Three possibilities $-A \operatorname{tracting} Node (\lambda_1 < \lambda_2 < 0)$ $+ Repulling Node (0, < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Node (0, < \lambda_1 < \lambda_2 < 0)$ $+ \operatorname{Repulling Spirit (0, < 0)$ $+ \operatorname{Repulling Node (0, < $	 ^{¬Note} the same sets tops are followed over if we have λ to be in term of λ. organization to the set of a same sharp are as W² peak, and therefore there will be not experiment that there are only and the same sets are as the same and the same set. We set <i>x</i>(<i>x</i>) the same of the main diagonal (Net <i>x</i>). The sign of <i>λ</i> gives its stability. In both cases, the sign of <i>λ</i> gives its stability. In both cases, the sign of <i>λ</i> gives its stability. In both cases, the sign of <i>λ</i> gives its stability. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> > 0, trajectories approach the origin parallel to <i>ξ</i>. If <i>λ</i> = 0, there exists a line of fixed points at the eigenvectors we transmost and smooth. 15.1 Properties of Phase Phane Trajectories cannot cross. When the prince functions <i>f</i> and <i>g</i> are continuous, trajectories are clumous and smooth. 15.2 Equilibrian Phase Portatis can have more than one, or none at all. To find a systee equilitria, solve <i>x'</i> and <i>y'</i> simultaneously. 15.4 Linet Cycle Allelines in this case are the same as before. 15.4 Linet Dyvin and parameting argument appendix exhibition to whore monthem by winding around more and marce closely from etimate or outside.

See (??).	on a, b conta	(Existence and Unique ining t ₀ . For any A and on (a, b) to the IVP y"-	$d B$ in \mathbb{R} , there exists	s a unique se
	A basis ex	ists for the general sec	ond order equation.	
		(Solution Space). The differential equation h		or a second
particular are tedious nethods can be easier	y'' + p(t)y	the real second order home y' + q(t)y = 0 (pt of a phase plane is identified on the real second order home)		
	exception of \dot{x}	replacing y.		
follow the following	13.3 Pro	operties of Eigen	values	
	Let \mathbf{A} be an	$n \times n$ matrix.		
	 λ is an 	eigenvalue of A if and	only if $ \mathbf{A} - \lambda \mathbf{I} = 0$	
nes. ng $(A - \lambda_i I) \vec{v}_i = 0$	 λ is an solution 	eigenvalue of A if and	only if $(\mathbf{A} - \lambda \mathbf{I})\vec{\mathbf{v}} = 0$	$\vec{\mathbf{j}}$ has a non-
	• A has a	zero eigenvalue if and	only if $ \mathbf{A} = 0$	
es larger than 2 or 3, scue!	• A and a	\mathbf{A}^T have the same char	acteristic polynomials	and eigenva
	13.4 Th	e Mind-Blowing	Part	
		Characteristic Roots (? lenced below.	?)? Well, they are id	<i>entical</i> to eig
ngular matrix (upper		linear second order dif	ferential equation:	
termined with λ^2 –		hat it has a characteris = $(r - 2)(r + 1) = 0$ of	tic equation of	
$(\operatorname{Tr}(\mathbf{A}^2) - \operatorname{Tr}^2(\mathbf{A})) -$	$[r_1, r_2] \begin{cases} 2 \\ - \end{cases}$	1		
	which create $u = c_1 e^{2t} + $	tes the general solutio $h cos^{-t}$	n of	
	In Section	?? we saw that we can	write a second order d	ifferential eq
s λ together with the ce.	as a system $\int \dot{x} = y'$			
linear transformation	y = 2y + which has	y' the matrix form $\vec{x}'_{-} = .$	A 3 -	
is a subspace of V .	$\vec{\mathbf{x}} = \begin{bmatrix} y \\ y' \end{bmatrix}$	and $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$		
n × n matrix. If ponding eigenvectors j independent vectors.		cteristic equation $ A - A$ te same eigenvalues as		
eigenvector, than that				
to be in terms of <i>i</i> . The		perties of Linear H h Distinct Eigenval		ential Equa
herefore there will be no		ential equation $\vec{x}' = A$	$\vec{\mathbf{x}}$ with distinct eigenvalue	alues, the foll
main diagonal.	properties ap	oply.		
t eigenvectors we call	Type	Eigenvalues	La Geometry	nearized System Stabilit
ie sign of λ gives its		$\lambda \in \lambda < 0$	Attracting Node	Asympt

 $\int \dot{x} = y$

12.2 Properties and Theorems

12.2 Properties and Theorems	12.3 Roots
For the linear homogeneous, second-order differential equation $y^{t} = y(t) + q(t) = 0$ with $p \ \text{ind} q \ \text{being}$ continuous functions of t , there exists a two dimensional vector space of solutions. Rewriting the above equation gives us $y^{t}(t) \equiv f(t, w_{1}) = -q(t)y^{t} = 0$	given second order dimerential equation and converting it into a quadratic equation, using which we can solve for the homogeneous solution.
which gives us the existence and uniqueness theorem for the second order equation.	$a\ddot{y} + b\dot{y} + cy = 0 \Leftrightarrow ar^2 + br + c = 0$ (24)
Theorem 2 (Existence and Uniqueness). Let $p(t)$ and $q(t)$ be continuous on a, b containing t_0 . For any A and B in \mathbb{R} , there exists a unique solution $y(t)$ defined on (a, b) to the IVP $y' + p(t)y' + q(t)y = 0$, $y(t_0) = A$, $y'(t_0) = B$	equations, there are three different possibilities for the solution:
A basis exists for the general second order equation.	 Two distinct real roots or zeros
Theorem 3 (Solution Space). The solution space S for a second order	One real root (a double root)
homogeneous differential equation has a Dimension of 2. For any linear second order homogeneous differential equation on (a, b),	Two imaginary roots
y'' + p(t)y' + q(t)y = 0 ² This concept of a phase plane is identical to the one introduced in (??) with the	These are summarized as follows. These methods allow us to generalize for higher order differential equa-
exception of \dot{x} replacing y .	tions and find solutions that would be otherwise impossible.
13.3 Properties of Eigenvalues	 The domain of the linear transformation is a vector space of vector functions.
Let A be an $n \times n$ matrix.	 The solution set is also a vector space of vector functions.
- λ is an eigenvalue of A if and only if $ \mathbf{A} - \lambda \mathbf{I} = 0$	•••••
• λ is an eigenvalue of A if and only if $(\mathbf{A} - \lambda \mathbf{I})\vec{\mathbf{v}} = \vec{0}$ has a non-trivial solution.	 The eigenspace for each eigenvalue is a one dimensional line in the direction of a vector in Rⁿ.
- A has a zero eigenvalue if and only if $ \mathbf{A} =0$	14 Linear Systems of Differential Equations
- ${\bf A}$ and ${\bf A}^T$ have the same characteristic polynomials and eigenvalues.	To define the linear first order differential equations system: An n -dimensional first order differential equations system on an open
13.4 The Mind-Blowing Part	interval ${\cal I}$ is one that can be written as a matrix vector equation.
Remember Characteristic Roots (??)? Well, they are <i>identical</i> to eigenval- ues as is evidenced below.	$\vec{x}'(t) = A(t)\vec{x}(t) + \vec{f}(t)$ (25)
Given the linear second order differential equation: y'' - y' - 2y = 0	 A(t) is an n × n matrix of continuous functions on I.
we know that it has a characteristic equation of	 f(t) is an n × 1 vector of continuous functions on I.
$r^2 - r - 2 = (r - 2)(r + 1) = 0$ with roots of	 \$\vec{x}(t)\$ is an \$n \times 1\$ solution vector.
$[r_1, r_2]$ $\begin{cases} 2 \\ -1 \end{cases}$	 If f(t) = 0, the system is homogeneous.
which creates the general solution of $u = c_1e^{2t} + c_2e^{-t}$	14.1 Graphical Methods
In Section ?? we saw that we can write a second order differential equation	We use the phase plane from before to accurately represent these systems.
as a system of equations: $\int \dot{x} = y'$	14.1.1 Nullclines
$\begin{cases} x - y \\ \dot{y} = 2y + y' \end{cases}$	The v nullcline is the set of all points with vertical slope which occur on
which has the matrix form $\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}}$: $\vec{\mathbf{x}} = \begin{bmatrix} y \\ y' \end{bmatrix}$ and $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ The characteristic equation $ \mathbf{A} - \lambda \mathbf{I} = 0$ for this matrix \mathbf{A} is $\lambda^2 - \lambda - 2 = 0$	The curve obtained by solving $x' = f(x, y) = 0$ The <i>h</i> nullkline is the same except with horizontal slope and is found with $y' = f(x, y) = 0$ At the intersection we get a fixed equilibrium point.

 $y(t_0) = A_0, y'(t_0) = A_1, \dots, y^{n-1}(t_0) = A_{n-1}$

eristic 14.1.2 Eigenvalues Eigenvalues play a large role in phase planes as well. For an autonomous ations and homogeneous system of differential linear system of equations:

12.1.3 Phase Planes For any autonomous second order differential equation x = P(x, z)for which p and q are continuous on (a, b), any two linearly independent solutions (y_0, y_0) form a basis of the solutions space S, and every solution y (a, b) can be written as $y(t) = c_{y(t)}(t) \rightarrow (c_t, c_b) \in \mathbb{R}$ for which direction given by $|U| = c_{y(t)}(t) \rightarrow (c_t, c_b) \in \mathbb{R}$ To generalize we can apply the same principle to *n*th order differential $|U| = \frac{1}{2}$

ned with directon given by $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 4 (Existence and Uniqueness for *nth* Order Dif- $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 4 (Existence and Uniqueness for *nth* Order Dif- $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 4 (Existence and Uniqueness for *nth* Order Dif- $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in $\begin{bmatrix} H - \frac{h}{q_{e}} \\ U - \frac{h}{q_{e}} \end{bmatrix}$ Theorem 5 and be found by parametrically combining the vectors in the vectors in the vector in the vect

 $\begin{aligned} & \int_{0}^{L} - y & y(t_{0}) = A_{0}, y(t_{0}$

12.3 Roots

Type	Eigenvalues	Linearized System 15		Nonlinear System	
		Geometry	Stability	Geometry	Stability
Real Distinct	$\lambda_1 < \lambda_2 < 0$	Attracting Node	Asymptotically Stable	Attracting Node	Asymptotically Stable
Roots	$0 < \lambda_2 < \lambda_1$	Repelling Node	Unstable	Repelling Node	Unstable
Roots	$\lambda_1 < 0 < \lambda_2$	Saddle	Unstable	Saddle	Unstable
Real Repeated	$\lambda_1 = \lambda_2 < 0$	Attracting Star of Degenerate	Asymptotically Stable	Attracting Node or Spiral	Asymptotically Stable
Roots		Node			
	$\lambda_1 = \lambda_2 > 0$	Repelling Star or Degenerate	Unstable	Repelling Node or Spiral	Unstable
		Node			
Complex	$\alpha > 0$	Repelling Spiral	Unstable	Repelling Spiral	Unstable
Conjugate	$\alpha < 0$	Attracting Spiral	Asymptotically Stable	Attracting Spiral	Asymptotically Stable
Roots	$\alpha = 0$	Center	Stable	Center or Spiral	Uncertain

18

Non-

Table 3: Table of Behavior Based on the System's Jacobian Matrix Eigenvalues

where f and g are twice differentiable, the linearized system at an equilibrium point (x_e,y_e) translated by $u=x-x_e$ and $v=y-y_e$ is

 $\begin{bmatrix} u \\ v \end{bmatrix}' = J(x_e, y_e)$ where $J(x_e, y_e) = \begin{bmatrix} f_x(x_e, y_e) & f_y(x_e, y_e) \\ g_x(x_e, y_e) & g_y(x_e, y_e) \end{bmatrix}$ (26)

Trajectories are toward or away based on the sign of the eigenvalue.
Along each eigenvector is the separatria that separates different curves